The mine car and its contents have a total mass of 6 Mg and a center of gravity at \( G \). If the coefficient of static friction between the wheels and the tracks is \( \mu_s = 0.4 \) when the wheels are locked, find the normal force acting on the front wheels at \( B \) and the rear wheels at \( A \) when the brakes at both \( A \) and \( B \) are locked. Does the car move?

**SOLUTION**

_Equations of Equilibrium:_ The normal reactions acting on the wheels at \( A \) and \( B \) are independent as to whether the wheels are locked or not. Hence, the normal reactions acting on the wheels are the same for both cases.

\[
\sum F_y = 0; \quad N_A (1.5) + 10(1.05) - 58.86(0.6) = 0
\]

\[N_A = 16.544 \text{ kN} = 16.5 \text{ kN}\]  

Ans.

\[
\sum M_B = 0; \quad N_A (1.5) + 10(1.05) - 58.86(0.6) = 0
\]

\[N_A = 16.544 \text{ kN} = 16.5 \text{ kN}\]  

Ans.

\[
\sum F_y = 0; \quad N_B + 16.544 - 58.86 = 0
\]

\[N_B = 42.316 \text{ kN} = 42.3 \text{ kN}\]  

Ans.

When both wheels at \( A \) and \( B \) are locked, then \( (F_A)_{\text{max}} = \mu_s N_A = 0.4(16.544) = 6.6176 \text{ kN} \) and \( (F_B)_{\text{max}} = \mu_s N_B = 0.4(42.316) = 16.9264 \text{ kN} \). Since \( (F_A)_{\text{max}} + (F_B)_{\text{max}} = 23.544 \text{ kN} > 10 \text{ kN} \), the wheels do not slip. Thus, the mine car does not move.  

Ans.
Determine the maximum force $P$ the connection can support so that no slipping occurs between the plates. There are four bolts used for the connection and each is tightened so that it is subjected to a tension of 4 kN. The coefficient of static friction between the plates is $\mu_s = 0.4$.

**SOLUTION**

**Free-Body Diagram:** The normal reaction acting on the contacting surface is equal to the sum total tension of the bolts. Thus, $N = 4(4) \text{ kN} = 16 \text{ kN}$. When the plate is on the verge of slipping, the magnitude of the friction force acting on each contact surface can be computed using the friction formula $F = \mu_s N = 0.4(16) \text{ kN}$. As indicated on the free-body diagram of the upper plate, $F$ acts to the right since the plate has a tendency to move to the left.

**Equations of Equilibrium:**

\[ \sum F_x = 0; \quad 0.4(16) - \frac{P}{2} = 0 \quad \text{Ans.} \]

\[ p = 12.8 \text{ kN} \]
8–3. If the coefficient of static friction at \( A \) is \( \mu = 0.4 \) and the collar at \( B \) is smooth so it only exerts a horizontal force on the pipe, determine the minimum distance \( x \) so that the bracket can support the cylinder of any mass without slipping. Neglect the mass of the bracket.

**Free Body Diagram.** The weight of cylinder tends to cause the bracket to slide downward. Thus, the frictional force \( F_A \) must act upwards as indicated in the free-body diagram shown in Fig. a. Here the bracket is required to be on the verge of slipping so that \( F_A = \mu N_A = 0.4 N_A \).

**Equations of Equilibrium.**

\[
\begin{align*}
+ \sum F_y &= 0; \quad 0.4 N_A - mg = 0 \\
+ \sum M_B &= 0; \quad 2.5 m g (0.2) + 0.4 (2.5 m g) (0.1) = m (g x - 0.1) \quad x = 0.5 \text{ m}
\end{align*}
\]

**Note.** Since \( x \) is independent of the mass of the cylinder, the bracket will not slip regardless of the mass of the cylinder provided \( x > 0.5 \text{ m} \).
If the coefficient of static friction at contacting surface between blocks A and B is \( \mu_s \), and that between block B and bottom is \( 2\mu_s \), determine the inclination \( \theta \) at which the identical blocks, each of weight \( W \), begin to slide.

Free - Body Diagram. Here, we will assume that the impending motion of the upper block is down the plane while the impending motion of the lower block is up the plane. Thus, the frictional force \( F \) acting on the upper block acts up the plane while the frictional forces \( F' \) acting on the lower block act down the plane as indicated on the free - body diagram of the upper and lower blocks shown in Figs. a and b, respectively. Since both blocks are required to be on the verge of slipping, then \( F = \mu_s N \) and \( F' = \mu_s N' \).

Equations of Equilibrium. Referring to Fig. a,

\[ \Sigma F_x = 0; \quad N - W \cos \theta = 0 \]
\[ \Sigma F_y = 0; \quad T + \mu_s (W \cos \theta) - W \sin \theta = 0 \]
\[ N = W \cos \theta \]
\[ T = W \sin \theta - \mu_s W \cos \theta \]

Using these results and referring to Fig. b,

\[ \Sigma F_x = 0; \quad N' - W \cos \theta - W \cos \theta = 0 \]
\[ \Sigma F_y = 0; \quad 2W \sin \theta - \mu_s W \cos \theta - \mu_s W \cos \theta - 2\mu_s (2W \cos \theta) - W \sin \theta = 0 \]
\[ \sin \theta - 2\mu_s \cos \theta = 0 \]

\[ \theta = \tan^{-1} \mu_s \]

Ans.

Since the analysis yields a positive \( \theta \), the above assumption is correct.
8–5. The coefficients of static and kinetic friction between the drum and brake bar are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. If $M = 50\text{ N} \cdot \text{m}$ and $P = 85\text{ N}$ determine the horizontal and vertical components of reaction at the pin $O$. Neglect the weight and thickness of the brake. The drum has a mass of 25 kg.

**Equations of Equilibrium**: From FBD (b),

$$+ \quad \Sigma M_O = 0 \quad 50 - F_y (0.125) = 0 \quad F_y = 400\text{ N}$$

From FBD (a),

$$+ \Sigma M_A = 0; \quad 85 (1.00) + 400 (0.5) - N_y (0.7) = 0 \quad N_y = 407.14\text{ N}$$

**Friction**: Since $F_y > (F_y)_{\text{max}} = \mu_s N_y = 0.4 (407.14) = 162.86\text{ N}$, the drum slips at point $B$ and rotates. Therefore, the coefficient of kinetic friction should be used. Thus, $F_y = \mu_s N_y = 0.3 N_y$.

**Equations of Equilibrium**: From FBD (b),

$$+ \Sigma M_A = 0; \quad 85 (1.00) + 0.3 N_y (0.5) - N_y (0.7) = 0 \quad N_y = 154.54\text{ N}$$

From FBD (b),

$$\uparrow \Sigma F_x = 0; \quad O_x - 245.25 = 154.54 = 0 \quad O_x = 400\text{ N} \quad \text{Ans.}$$

$$\uparrow \Sigma F_x = 0; \quad 0.3 (154.54) - O_y = 0 \quad O_y = 46.4\text{ N} \quad \text{Ans.}$$
The coefficient of static friction between the drum and brake bar is \( \mu_s = 0.4 \). If the moment \( M = 35 \text{ N} \cdot \text{m} \), determine the smallest force \( P \) that needs to be applied to the brake bar in order to prevent the drum from rotating. Also determine the corresponding horizontal and vertical components of reaction at pin \( O \). Neglect the weight and thickness of the brake bar. The drum has a mass of 25 kg.

**Equations of Equilibrium:** From FBD (b).

\[
\sum M_O = 0 \quad 35 - F_b (0.125) = 0 \quad F_b = 280 \text{ N}
\]

From FBD (a).

\[
\sum F_x = 0; \quad P (1.00) + 280 (0.5) - N_B (0.7) = 0
\]

**Friction:** When the drum is on the verge of rotating,

\[
F_b = \mu_s N_B \\
F_b = 0.4 \times 280 \\
F_b = 700 \text{ N}
\]

Substituting \( N_B = 700 \text{ N} \) into Eq. (1) yields

\[
P = 350 \text{ N}
\]
The block brake consists of a pin-connected lever and friction block at \( B \). The coefficient of static friction between the wheel and the lever is \( \mu_s = 0.3 \), and a torque of \( 5 \text{ N} \cdot \text{m} \) is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) \( P = 30 \text{ N} \), (b) \( P = 70 \text{ N} \).

**SOLUTION**

To hold lever:

\[
\sum \tau_B = 0; \quad F_B(0.15) - 5 = 0; \quad F_B = 33.333 \text{ N}
\]

Require

\[
N_B = \frac{33.333 \text{ N}}{0.3} = 111.1 \text{ N}
\]

Lever,

\[
\sum \tau_A = 0; \quad P_{\text{reqd}}(0.6) - 111.1(0.2) - 33.333(0.05) = 0
\]

\( P_{\text{reqd}} = 39.8 \text{ N} \)

(a) \( P = 30 \text{ N} < 39.8 \text{ N} \) **No**

(b) \( P = 70 \text{ N} > 39.8 \text{ N} \) **Yes**

An.
8–8. The block brake consists of a pin-connected lever and friction block at B. The coefficient of static friction between the wheel and the lever is \( \mu_s = 0.3 \), and a torque of 5 N \( \cdot \) m is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) \( P = 30 \) N, (b) \( P = 70 \) N.

**SOLUTION**

To hold lever:

\[
\zeta + \sum M_O = 0; \quad -F_B(0.15) + 5 = 0; \quad F_B = 33.333 \text{ N}
\]

Require

\[
N_B = \frac{33.333 \text{ N}}{0.3} = 111.1 \text{ N}
\]

Lever,

\[
\zeta + \sum M_A = 0; \quad P_{\text{Reqt}} (0.6) - 111.1(0.2) + 33.333(0.05) = 0
\]

\[
P_{\text{Reqt}} = 34.26 \text{ N}
\]

a) \( P = 30 \text{ N} < 34.26 \text{ N} \) No

b) \( P = 70 \text{ N} > 34.26 \text{ N} \) Yes
The uniform hoop of weight \( W \) is suspended from the peg at \( A \) and a horizontal force \( P \) is slowly applied at \( B \). If the hoop begins to slip at \( A \) when the angle is \( \theta \), determine the coefficient of static friction between the hoop and the peg.

Given:
\[ \theta = 30 \text{ deg} \]

Solution:
\[
\begin{align*}
\Sigma F_x &= 0; \quad \mu N_A \cos(\theta) + P - N_A \sin(\theta) = 0 \\
&= P = (\mu \cos(\theta) - \sin(\theta))N_A \\
\Sigma F_y &= 0; \quad \mu N_A \sin(\theta) - W + N_A \cos(\theta) = 0 \\
&= W = (\mu \sin(\theta) + \cos(\theta))N_A \\
\Sigma M_A &= 0; \quad -W r \sin(\theta) + P \left(r + r \cos(\theta)\right) = 0 \\
&= W \sin(\theta) = P(1 + \cos(\theta)) \\
(\mu \sin(\theta) + \cos(\theta))\sin(\theta) &= (\sin(\theta) - \mu \cos(\theta))(1 + \cos(\theta)) \\
\mu &= \frac{\sin(\theta)}{1 + \cos(\theta)} \quad \text{Ans.} \\
&= \mu = 0.27
\end{align*}
\]
The uniform hoop of weight $W$ is suspended from the peg at $A$ and a horizontal force $P$ is slowly applied at $B$. If the coefficient of static friction between the hoop and peg is $\mu_s$, determine if it is possible for the hoop to reach an angle $\theta$ before the hoop begins to slip.

Given:

$\mu_s = 0.2$

$\theta = 30 \text{ deg}$

Solution:

$\Sigma F_x = 0; \quad \mu N_A \cos(\theta) + P - N_A \sin(\theta) = 0$

$P = (\mu \cos(\theta) - \sin(\theta)) N_A$

$\Sigma F_y = 0; \quad \mu N_A \sin(\theta) - W + N_A \cos(\theta) = 0$

$W = (\mu \sin(\theta) + \cos(\theta)) N_A$

$\Sigma M_A = 0; \quad -W r \sin(\theta) + P(r + r \cos(\theta)) = 0$

$W \sin(\theta) = P(1 + \cos(\theta))$

$(\mu \sin(\theta) + \cos(\theta)) \sin(\theta) = (\sin(\theta) - \mu \cos(\theta))(1 + \cos(\theta))$

$\mu = \frac{\sin(\theta)}{1 + \cos(\theta)} \quad \mu = 0.27$

If $\mu_s = 0.20 < \mu = 0.27$ then it is not possible to reach $\theta = 30.00 \text{ deg}$.  

Answ.
The fork lift has a weight $W_1$ and center of gravity at $G$. If the rear wheels are powered, whereas the front wheels are free to roll, determine the maximum number of crates, each of weight $W_2$ that the fork lift can push forward. The coefficient of static friction between the wheels and the ground is $\mu_s$ and between each crate and the ground is $\mu'_s$.

**Given:**

$$W_1 := 12 \text{kN}$$
$$W_2 := 150 \cdot (9.81) \text{ N}$$
$$\mu_s := 0.4$$
$$\mu'_s := 0.35$$
$$a := 0.75 \text{ m}$$
$$b := 0.375 \text{ m}$$
$$c := 1.05 \text{ m}$$

**Solution:**

Fork lift:

$$\Sigma M_B = 0; \quad W_1 \cdot c - N_A \cdot (b + c) = 0$$

$$N_A := \frac{W_1 \cdot c}{b + c} \quad N_A = 8.8 \text{ kN}$$

$$\Sigma F_x = 0; \quad \mu_s \cdot N_A - P = 0$$

$$P := \mu_s \cdot N_A \quad P = 3.54 \text{ kN}$$

Crate:

$$\Sigma F_y = 0; \quad N_c - W_2 = 0$$

$$N_c := W_2 \quad N_c = 1.471 \text{ kN}$$

$$\Sigma F_x = 0; \quad P' - \mu'_s \cdot N_c = 0$$

$$P' := \mu'_s \cdot N_c \quad P' = 0.515 \text{ kN}$$

Thus

$$n := \frac{P}{P'} \quad n = 6.87 \quad n := \text{ floor}(n) \quad n = 6 \quad \text{Ans.}$$
8–12.

If a torque of $M = 300 \text{ N} \cdot \text{m}$ is applied to the flywheel, determine the force that must be developed in the hydraulic cylinder $CD$ to prevent the flywheel from rotating. The coefficient of static friction between the friction pad at $B$ and the flywheel is $\mu_s = 0.4$.

**SOLUTION**

**Free-Body Diagram:** First we will consider the equilibrium of the flywheel using the free-body diagram shown in Fig. a. Here, the frictional force $F_B$ must act to the left to produce the counterclockwise moment opposing the impending clockwise rotational motion caused by the 300 N \cdot m couple moment. Since the wheel is required to be on the verge of slipping, then $F_B = \mu_s N_B = 0.4 N_B$. Subsequently, the free-body diagram of member $ABC$ shown in Fig. b will be used to determine $F_{CD}$.

**Equations of Equilibrium:** We have

\[ \zeta + \sum M_O = 0; \quad 0.4 N_B(0.3) - 300 = 0 \quad N_B = 2500 \text{ N} \]

Using this result,

\[ \zeta + \sum M_A = 0; \quad F_{CD} \sin 30^\circ(1.6) + 0.4(2500)(0.06) - 2500(1) = 0 \]

\[ F_{CD} = 3050 \text{ N} = 3.05 \text{ kN} \quad \text{Ans.} \]
8–13.

The cam is subjected to a couple moment of 5 N·m. Determine the minimum force $P$ that should be applied to the follower in order to hold the cam in the position shown. The coefficient of static friction between the cam and the follower is $\mu_s = 0.4$. The guide at $A$ is smooth.

**SOLUTION**

Cam:

$$\zeta + \Sigma M_O = 0; \quad 5 - 0.4 N_B (0.06) - 0.01 (N_B) = 0$$

$$N_B = 147.06 \text{ N}$$

Follower:

$$+ \Sigma F_y = 0; \quad 147.06 - P = 0$$

$$P = 147 \text{ N} \quad \text{Ans.}$$
8.14. A 35-kg disk rests on an inclined surface for which \( \mu_s = 0.2 \). Determine the maximum vertical force \( P \) that may be applied to link \( AB \) without causing the disk to slip at \( C \).

**Equations of Equilibrium**: From FBD (a).

\[ \sum F_y = 0; \quad P(600) - A_y(900) = 0 \quad A_y = 0.6667P \]

From FBD (b).

\[ \sum F_x = 0; \quad N_c \sin 60^\circ - F_c \sin 30^\circ - 0.6667P - 343.35 = 0 \quad [1] \]

\[ \sum M_C = 0; \quad F_c(200) - 0.6667P(200) = 0 \quad [2] \]

**Friction**: If the disk is on the verge of moving, slipping would have to occur at point \( C \). Hence, \( F_c = \mu_s N_c = 0.2 N_c \). Substituting this value into Eqs. [1] and [2] and solving, we have

\[ P = 182 \text{ N} \]

\[ N_c = 606.60 \text{ N} \]

\[ A_y = 0.6667P \]

\[ 35(9.81) + 343.35 = 35(9.81) + 343.35 \]
8–15.

The car has a mass of 1.6 Mg and center of mass at $G$. If the coefficient of static friction between the shoulder of the road and the tires is $\mu_s = 0.4$, determine the greatest slope $\theta$ the shoulder can have without causing the car to slip or tip over if the car travels along the shoulder at constant velocity.

**SOLUTION**

**Tipping:**

$\sum \mathcal{M}_A = 0; \quad -W \cos \theta(0.75) + W \sin \theta(0.75) = 0$

$\tan \theta = 1$

$\theta = 45^\circ$

**Slipping:**

$\sum \mathcal{F}_x = 0; \quad 0.4 N - W \sin \theta = 0$

$\sum \mathcal{F}_y = 0; \quad N - W \cos \theta = 0$

$\tan \theta = 0.4$

$\theta = 21.8^\circ$  

**Ans.** (Car slips before it tips.)
8–16. If the coefficient of static friction between the collars \( A \) and \( B \) and the rod is \( \mu_s = 0.6 \), determine the maximum angle \( \theta \) for the system to remain in equilibrium, regardless of the weight of cylinder \( D \). Links \( AC \) and \( BC \) have negligible weight and are connected together at \( C \) by a pin.

**Free-Body Diagram.** Due to the symmetrical loading and system, collars \( A \) and \( B \) will slip simultaneously. Thus, it is sufficient to consider the equilibrium of either collar. Here, the equilibrium of collar \( B \) will be considered. Since collar \( B \) is required to be on the verge of sliding down the rod the friction force \( F_B \) must act up the rod such that \( F_B = \mu_s N_B = 0.6 N_B \) as indicated on the free-body diagram of the collar shown in Fig. \( a \).

**Equations of Equilibrium.**

\[ \sum F_y = 0; \quad N_B - F_B \sin(75^\circ - \theta) = 0 \quad N_B = F_B \sin(75^\circ - \theta) \]

\[ \sum F_x = 0; \quad 0.6[F_B \sin(75^\circ - \theta)] - F_B \cos(75^\circ - \theta) = 0 \]

\[ \tan(75^\circ - \theta) = 1.6667 \]

\[ \theta = 16.0^\circ \quad \text{Ans.} \]
8–17. If $\theta = 15^\circ$, determine the minimum coefficient of static friction between the collars $A$ and $B$ and the rod required for the system to remain in equilibrium, regardless of the weight of cylinder $D$. Links $AC$ and $BC$ have negligible weight and are connected together at $C$ by a pin.

Free - Body Diagram. Due to the symmetrical loading and system, collars $A$ and $B$ will slip simultaneously. Thus, it is sufficient to consider the equilibrium of either collar. Here, the equilibrium of collar $B$ will be considered. Since collar $B$ is required to be on the verge of sliding down the rod the friction force $F_B$ must up the rod such that $F_B = \mu_s N_B = 0.6N_B$ as indicated on the free - body diagram of the collar shown in Fig. a.

Equations of Equilibrium.

$$\Sigma F_y = 0: \quad N_B - F_{BC} \sin 60^\circ = 0 \quad N_B = 0.8660F_{BC}$$

$$\Sigma F_x = 0: \quad \mu_s [0.8660F_{BC}] - F_{BC} \cos 60^\circ = 0 \quad \mu_s = 0.577$$

Ans.
The 5-kg cylinder is suspended from two equal-length cords. The end of each cord is attached to a ring of negligible mass that passes along a horizontal shaft. If the rings can be separated by the greatest distance \( d = 400 \text{ mm} \) and still support the cylinder, determine the coefficient of static friction between each ring and the shaft.

**SOLUTION**

**Equilibrium of the Cylinder:** Referring to the FBD shown in Fig. a,

\[
+ \sum F_y = 0; \quad 2 \left[ T \left( \frac{\sqrt{32}}{6} \right) \right] - m(9.81) = 0 \quad T = 5.2025 \text{ m}
\]

**Equilibrium of the Ring:** Since the ring is required to be on the verge to slide, the frictional force can be computed using friction formula \( F_f = \mu N \) as indicated in the FBD of the ring shown in Fig. b. Using the result of \( I \),

\[
+ \sum F_y = 0; \quad N - 5.2025 m \left( \frac{\sqrt{32}}{6} \right) = 0 \quad N = 4.905 \text{ m}
\]

\[
\sum F_x = 0; \quad \mu(4.905\ m) - 5.2025\ m \left( \frac{2}{6} \right) = 0
\]

\[
\mu = \frac{3.54}{5.2025} = 0.354 \quad \text{Ans.}
\]
8–19.

The 5-kg cylinder is suspended from two equal-length cords. The end of each cord is attached to a ring of negligible mass, which passes along a horizontal shaft. If the coefficient of static friction between each ring and the shaft is \( \mu_s = 0.5 \), determine the greatest distance \( d \) by which the rings can be separated and still support the cylinder.

**SOLUTION**

**Friction:** When the ring is on the verge to sliding along the rod, slipping will have to occur. Hence, \( F = \mu N = 0.5N \). From the force diagram \( (T \text{ is the tension developed by the cord}) \)

\[
\tan \theta = \frac{N}{0.5N} = 2 \quad \theta = 63.43^\circ
\]

**Geometry:**

\[
d = 2(600 \cos 63.43^\circ) = .537 \text{ mm}
\]

Ans.
8–20. The pipe is hoisted using the tongs. If the coefficient of static friction at A and B is $\mu_s$, determine the smallest dimension $b$ so that any pipe of inner diameter $d$ can be lifted.

Require:

- \[ F_s = \frac{W}{2} \leq \mu_s N_s \]
- \[ \sum F = 0 ; \quad -\frac{W}{2}(\frac{d}{2}) - N_s(h) + \frac{W}{2} = 0 \]

\[ N_s = \frac{W}{2h}(b - \frac{d}{2}) \]

Thus,

\[ \frac{W}{2} \leq \frac{\mu_s W}{2h}(b - \frac{d}{2}) \]

\[ h \leq (b - \frac{d}{2})\mu_s \]

\[ b \geq \frac{h}{\mu_s} + \frac{d}{2} \]

\[ b = \frac{h}{\mu_s} + \frac{d}{2} \quad \text{Ans} \]
The uniform pole has a weight $W$ and length $L$. Its end $B$ is tied to a supporting cord, and end $A$ is placed against the wall, for which the coefficient of static friction is $\mu_s$. Determine the largest angle $\theta$ at which the pole can be placed without slipping.

**SOLUTION**

$$\zeta + \Sigma M_B = 0; \quad -N_A (L \cos \theta) - \mu_s N_A (L \sin \theta) + W \left(\frac{L}{2} \sin \theta\right) = 0 \quad (1)$$

$$\sum F_x = 0; \quad N_A - T \sin \frac{\theta}{2} = 0 \quad (2)$$

$$+ \sum F_y = 0; \quad \mu_s N_A - W + T \cos \frac{\theta}{2} = 0 \quad (3)$$

Substitute Eq. (2) into Eq. (3): $\mu_s T \sin \frac{\theta}{2} - W + T \cos \frac{\theta}{2} = 0$

$$W = T \left(\cos \frac{\theta}{2} + \mu_s \sin \frac{\theta}{2}\right) \quad (4)$$

Substitute Eqs. (2) and (3) into Eq. (1):

$$T \sin \frac{\theta}{2} \cos \theta - T \cos \frac{\theta}{2} \sin \theta + \frac{W}{2} \sin \theta = 0 \quad (5)$$

Substitute Eq. (4) into Eq. (5):

$$\sin \frac{\theta}{2} \cos \theta - \cos \frac{\theta}{2} \sin \theta + \frac{1}{2} \cos \frac{\theta}{2} \sin \theta + \frac{1}{2} \mu_s \sin \frac{\theta}{2} \sin \theta = 0$$

$$-\sin \frac{\theta}{2} + \frac{1}{2} \left(\cos \frac{\theta}{2} + \mu_s \sin \frac{\theta}{2}\right) \sin \theta = 0$$

$$\cos \frac{\theta}{2} + \mu_s \sin \frac{\theta}{2} = \frac{1}{\cos \frac{\theta}{2}}$$

$$\cos^2 \frac{\theta}{2} + \mu_s \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 1$$

$$\mu_s \sin \frac{\theta}{2} \cos \frac{\theta}{2} = \sin \frac{\theta}{2}$$

$$\tan \frac{\theta}{2} = \mu_s$$

$$\theta = 2 \tan^{-1} \mu_s \quad \text{Ans.}$$

Also, because we have a three–force member,

$$\frac{L}{2} = \frac{L}{2} \cos \theta + \tan \phi \left(\frac{L}{2} \sin \theta\right)$$

$$1 = \cos \theta + \mu_s \sin \theta$$

$$\mu_s = \frac{1 - \cos \theta}{\sin \theta} = \tan \frac{\theta}{2}$$

$$\theta = 2 \tan^{-1} \mu_s \quad \text{Ans.}$$
If the clamping force is $F = 200$ N and each board has a mass of 2 kg, determine the maximum number of boards the clamp can support. The coefficient of static friction between the boards is $\mu_s = 0.3$, and the coefficient of static friction between the boards and the clamp is $\mu_s' = 0.45$.

**SOLUTION**

**Free-Body Diagram:** The boards could be on the verge of slipping between the two boards at the ends or between the clamp. Let $n$ be the number of boards between the clamp. Thus, the number of boards between the two boards at the ends is $n - 2$. If the boards slip between the two end boards, then $F = \mu_s N = 0.3(200) = 60$ N.

**Equations of Equilibrium:** Referring to the free-body diagram shown in Fig. a, we have

$$+ \uparrow \Sigma F_y = 0; \quad 2(60) - (n - 2)(9.81) = 0 \quad n = 8.12$$

If the end boards slip at the clamp, then $F' = \mu_s' N = 0.45(200) = 90$ N. By referring to the free-body diagram shown in Fig. b, we have

$$\zeta + \uparrow \Sigma F_y = 0; \quad 2(90) - n(2)(9.81) = 0 \quad n = 9.17$$

Thus, the maximum number of boards that can be supported by the clamp will be the smallest value of $n$ obtained above, which gives

$$n = 8$$

Ans.
A 35-kg disk rests on an inclined surface for which \( \mu_s = 0.3 \). Determine the maximum vertical force \( P \) that may be applied to link \( AB \) without causing the disk to slip at \( C \).

**SOLUTION**

*Equations of Equilibrium:* From FBD (a),

\[ \sum F_y = 0; \quad P(600) - A_y(900) = 0 \quad A_y = 0.6667P \]

From FBD (b),

\[ + \sum F_y = 0 \quad N_C \sin 60^\circ - F_C \sin 30^\circ - 0.6667P - 343.35 = 0 \quad (1) \]

\[ \sum F_x = 0; \quad F_C(200) - 0.6667P(200) = 0 \quad (2) \]

*Friction:* If the disk is on the verge of moving, slipping would have to occur at point \( C \). Hence, \( F_C = \mu_s N_C = 0.3N_C \). Substituting this value into Eqs. (1) and (2) and solving, we have

\[ P = 371.4 \text{ N} \quad \text{Ans.} \]

\[ N_C = 825.3 \text{ N} \]
8-24. The coefficient of static friction between the shoes at A and B of the tongs and the pallet is \( \mu_s = 0.5 \), and between the pallet and the floor \( \mu_s = 0.4 \). If a horizontal towing force of \( P = 300 \text{ N} \) is applied to the tongs, determine the largest mass that can be towed.

**Chain:**
\[ + \sum F_x = 0; \quad 2F \sin 60^\circ - 300 = 0 \]
\[ F = 175.2 \text{ N} \]

**Tongs:**
\[ + \sum F_x = 0; \quad -173.2 \cos 60^\circ (75) - 173.2 \sin 60^\circ (50) + N_A (75) - F_s (20) = 0 \]
\[ F_s = 0.5N_A \]
\[ F_s = 107.7 \text{ N} \]

**Crate:**
\[ - \sum F_x = 0; \quad F = 2(107.7) = 215.3 \text{ N} \]
\[ F = \mu N; \quad F = 0.4N \]
\[ N = 538.3 \text{ N} \]

\[ + \sum F_x = 0; \quad W = 538.3 \text{ N} \]
\[ m = \frac{538.3}{9.81} = 54.9 \text{ kg} \]
The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment $M_0$. If the coefficient of static friction between the wheel and the block is $\mu_s$, determine the smallest force $P$ that should be applied.

Solution:

\[ \sum M_C = 0; \quad P \ a - N \ b + \mu_s \ N c = 0 \]

\[ N = \frac{P \ a}{b - \mu_s \ c} \]

\[ \sum M_O = 0; \quad \mu_s \ N r - M_0 = 0 \]

\[ \mu_s \ P \ a \ r \]

\[ = \frac{M_0}{b - \mu_s \ c} \]

\[ P = \frac{M_0 \ (b - \mu_s \ c)}{\mu_s \ r \ a} \]

Ans.

The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment $M_0$. If the coefficient of static friction between the wheel and the block is $\mu_s$, show that the brake is self locking, i.e., $P \leq 0$, provided $\frac{b}{c} \leq \mu_s$.

Solution:

1. $\sum M_c = 0; \quad P \ a - N \ b + \mu_s \ N \ c = 0$

   $$N = \frac{P \ a}{b - \mu_s \ c}$$

2. $\sum M_O = 0; \quad \mu_s \ N \ r - M_O = 0$

   $$\frac{\mu_s \ P \ a \ r}{b - \mu_s \ c} = M_O$$

   $$P = \frac{M_O (b - \mu_s \ c)}{\mu_s \ r \ a}$$

   $P < 0$ if $(b - \mu_s \ c) < 0$ i.e., if $\frac{b}{c} < \mu_s$
8.27.

The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment $M_0$. If the coefficient of static friction between the wheel and the block is $\mu_s$, determine the smallest force $P$ that should be applied if the couple moment $M_0$ is applied counterclockwise.

Solution:

\[ \Sigma M_C = 0; \quad P \ a - N \ b - \mu_s \ N \ c = 0 \]

\[ N = \frac{P \ a}{b + \mu_s \ c} \]

\[ \Sigma M_O = 0; \quad -\mu_s \ N \ r + M_O = 0 \]

\[ \frac{\mu_s \ P \ a \ r}{b + \mu_s \ c} = M_O \]

\[ P = \frac{M_O (b + \mu_s \ c)}{\mu_s \ r \ a} \]  

Ans.
8-28. The 10-kg cylinder is suspended from two equal-length cords. The end of each cord is attached to a ring of negligible mass, which passes along a horizontal shaft. If the coefficient of static friction between each ring and the shaft is $\mu_s = 0.5$, determine the greatest distance $d$ by which the rings can be separated and still support the cylinder.

**Friction**: When the ring is on the verge to sliding along the rod, slipping will have to occur. Hence, $F = \mu N = 0.5N$. From the force diagram ($T$ is the tension developed by the cord):

$$\tan \theta = \frac{N}{0.5N} = 2 \quad \theta = 63.43^\circ$$

**Geometry**:

$$d = 2(600 \cos 63.43^\circ) = 537 \text{ mm}$$
The friction pawl is pinned at $A$ and rests against the wheel at $B$. It allows freedom of movement when the wheel is rotating counterclockwise about $C$. Clockwise rotation is prevented due to friction of the pawl which tends to bind the wheel. If $(\mu_s)_B = 0.6$, determine the design angle $\theta$ which will prevent clockwise motion for any value of applied moment $M$. \textit{Hint}: Neglect the weight of the pawl so that it becomes a two-force member.

\textbf{SOLUTION}

\textit{Friction}: When the wheel is on the verge of rotating, slipping would have to occur. Hence, $F_B = \mu N_B = 0.6 N_B$. From the force diagram ($F_{AB}$ is the force developed in the two force member $AB$)

\[
\tan(20^\circ + \theta) = \frac{0.6N_B}{N_B} = 0.6
\]

\[
\theta = 11.0^\circ \text{ \ (Ans.)}
\]
If $\theta = 30^\circ$ determine the minimum coefficient of static friction at $A$ and $B$ so that equilibrium of the supporting frame is maintained regardless of the mass of the cylinder C. Neglect the mass of the rods.

**SOLUTION**

**Free-Body Diagram:** Due to the symmetrical loading and system, ends $A$ and $B$ of the rod will slip simultaneously. Since end $B$ tends to move to the right, the friction force $F_B$ must act to the left as indicated on the free-body diagram shown in Fig. a.

**Equations of Equilibrium:** We have

\[ \sum F_x = 0; \quad F_{BC} \sin 30^\circ - F_B = 0 \]
\[ + \sum F_y = 0; \quad N_B - F_{BC} \cos 30^\circ = 0 \]

Therefore, to prevent slipping the coefficient of static friction ends $A$ and $B$ must be at least

\[ \mu_s = \frac{F_B}{N_B} = \frac{0.5F_{BC}}{0.8660F_{BC}} = 0.577 \]

Ans.
If the coefficient of static friction at A and B is \( \mu_s = 0.6 \), determine the maximum angle \( \theta \) so that the frame remains in equilibrium, regardless of the mass of the cylinder. Neglect the mass of the rods.

**SOLUTION**

**Free-Body Diagram:** Due to the symmetrical loading and system, ends A and B of the rod will slip simultaneously. Since end B is on the verge of sliding to the right, the friction force \( F_B \) must act to the left such that as indicated on the free-body diagram shown in Fig. a.

**Equations of Equilibrium:** We have

\[ + \sum F_y = 0; \quad N_B - F_{BC} \cos \theta = 0 \]
\[ \sum F_x = 0; \quad F_{BC} \sin \theta - 0.6(F_{BC} \cos \theta) = 0 \]
\[ \tan \theta = 0.6 \]
\[ \theta = 31.0^\circ \]

Ans. \( \theta = 31.0^\circ \)
8–32.

The semicylinder of mass $m$ and radius $r$ lies on the rough inclined plane for which $\phi = 10^\circ$ and the coefficient of static friction is $\mu_s = 0.3$. Determine if the semicylinder slides down the plane, and if not, find the angle of tip $\theta$ of its base $AB$.

**SOLUTION**

*Equations of Equilibrium:*

\[ \sum F_x = 0; \quad F(r) - 9.81m \sin \left( \frac{4r}{3\pi} \right) = 0 \]  
\[ \sum F_y = 0; \quad F \cos 10^\circ - N \sin 10^\circ = 0 \]  
\[ + \sum F_z = 0 \quad F \sin 10^\circ + N \cos 10^\circ - 9.81m = 0 \]

Solving Eqs. (1), (2) and (3) yields

\[ N = 9.661m \quad F = 1.703m \]

\[ \theta = 24.2^\circ \]

*Friction:* The maximum friction force that can be developed between the semicylinder and the inclined plane is $(F)_{max} = \mu_s N = 0.3(9.661m) = 2.898m$. Since $F_{max} > F = 1.703m$, the semicylinder will not slide down the plane. Ans.
The semicylinder of mass \( m \) and radius \( r \) lies on the rough inclined plane. If the inclination \( \theta = 15^\circ \), determine the smallest coefficient of static friction which will prevent the semicylinder from slipping.

**SOLUTION**

*Equations of Equilibrium:*

\[ +\sum F_x = 0; \quad F - 9.81m \sin 15^\circ = 0 \quad F = 2.539m \]

\[ \sum F_y = 0; \quad N - 9.81m \cos 15^\circ = 0 \quad N = 9.476m \]

*Friction:* If the semicylinder is on the verge of moving, slipping would have to occur. Hence,

\[ F = \mu_s N \]

\[ 2.539m = \mu_s (9.476m) \]

\[ \mu_s = 0.268 \]

Ans.
The coefficient of static friction between the 150-kg crate and the ground is $\mu_s = 0.3$, while the coefficient of static friction between the 80-kg man’s shoes and the ground is $\mu_s' = 0.4$. Determine if the man can move the crate.

**SOLUTION**

**Free-Body Diagram:** Since $P$ tends to move the crate to the right, the frictional force $F_C$ will act to the left as indicated on the free-body diagram shown in Fig. a. Since the crate is required to be on the verge of sliding the magnitude of $F_C$ can be computed using the friction formula, i.e. $F_C = \mu_s N_C = 0.3 N_C$. As indicated on the free-body diagram of the man shown in Fig. b, the frictional force $F_m$ acts to the right since force $P$ has the tendency to cause the man to slip to the left.

**Equations of Equilibrium:** Referring to Fig. a,

\[ + \uparrow \Sigma F_y = 0; \quad N_C + P \sin 30^\circ - 150(9.81) = 0 \]
\[ - \Sigma F_x = 0; \quad P \cos 30^\circ - 0.3 N_C = 0 \]

Solving,

\[ P = 434.49 \text{ N} \]
\[ N_C = 1254.26 \text{ N} \]

Using the result of $P$ and referring to Fig. b, we have

\[ + \uparrow \Sigma F_y = 0; \quad N_m - 434.49 \sin 30^\circ - 80(9.81) = 0 \quad N_m = 1002.04 \text{ N} \]
\[ - \Sigma F_x = 0; \quad F_m - 434.49 \cos 30^\circ = 0 \quad F_m = 376.28 \text{ N} \]

Since $F_m < F_{\text{max}} = \mu_s' N_m = 0.4(1002.04) = 400.82 \text{ N}$, the man does not slip. Thus, he can move the crate. **Ans.**
If the coefficient of static friction between the crate and the ground is $\mu_s = 0.3$, determine the minimum coefficient of static friction between the man’s shoes and the ground so that the man can move the crate.

**SOLUTION**

**Free-Body Diagram:** Since force $\mathbf{P}$ tends to move the crate to the right, the frictional force $\mathbf{F}_C$ will act to the left as indicated on the free-body diagram shown in Fig. a. Since the crate is required to be on the verge of sliding, $F_C = \mu_s N_C = 0.3 N_C$. As indicated on the free-body diagram of the man shown in Fig. b, the frictional force $\mathbf{F}_m$ acts to the right since force $\mathbf{P}$ has the tendency to cause the man to slip to the left.

**Equations of Equilibrium:** Referring to Fig. a,

\[
\begin{align*}
\hat{x} \sum F &= 0; \quad F_C - \mu_s N_C = 0 \\
\hat{y} \sum F &= 0; \quad N_C + P \sin 30^\circ - 150 \sin(9.81) = 0
\end{align*}
\]

Solving yields

\[
\begin{align*}
P &= 434.49 \text{ N} \\
N_C &= 1245.26 \text{ N}
\end{align*}
\]

Using the result of $P$ and referring to Fig. b,

\[
\begin{align*}
\hat{x} \sum F &= 0; \quad F_m - 434.49 \cos 30^\circ - 80 \cos(9.81) = 0 \\
\hat{y} \sum F &= 0; \quad N_m + P \sin 30^\circ - 150 \sin(9.81) = 0
\end{align*}
\]

Thus, the required minimum coefficient of static friction between the man’s shoes and the ground is given by

\[
\mu_s' = \frac{F_m}{N_m} = \frac{376.28}{1002.04} = 0.376
\]

Ans.
The thin rod has a weight $W$ and rests against the floor and wall for which the coefficients of static friction are $\mu_A$ and $\mu_B$, respectively. Determine the smallest value of $\theta$ for which the rod will not move.

**SOLUTION**

*Equations of Equilibrium:*

1. $\sum F_x = 0; \quad F_A - N_B = 0 \quad (1)$
2. $\sum F_y = 0 \quad N_A + F_B - W = 0 \quad (2)$
3. $\sum M_A = 0; \quad N_B (L \sin \theta) + F_B (\cos \theta)L - W \cos \theta \left( \frac{L}{2} \right) = 0 \quad (3)$

*Friction:* If the rod is on the verge of moving, slipping will have to occur at points $A$ and $B$. Hence, $F_A = \mu_A N_A$ and $F_B = \mu_B N_B$. Substituting these values into Eqs. (1), (2), and (3) and solving we have

$$N_A = \frac{W}{1 + \mu_A \mu_B}, \quad N_B = \frac{\mu_A W}{1 + \mu_A \mu_B}$$

$$\theta = \tan^{-1} \left( \frac{1 - \frac{\mu_A \mu_B}{2 \mu_A}}{1 + \mu_A \mu_B} \right)$$

Ans.
Determine the magnitude of force $P$ needed to start towing the crate of mass $M$. Also determine the location of the resultant normal force acting on the crate, measured from point $A$.

Given:

$$M = 40 \text{ kg} \quad c = 200 \text{ mm}$$
$$\mu_s = 0.3 \quad d = 3$$
$$a = 400 \text{ mm} \quad e = 4$$
$$b = 800 \text{ mm}$$

Solution:

Initial guesses: 

$$N_C = 200 \text{ N} \quad P = 50 \text{ N}$$

Given

$$\Sigma F_x = 0; \quad \left( \frac{d}{\sqrt{d^2 + e^2}} \right) P - \mu_s N_C = 0$$

$$\Sigma F_y = 0; \quad N_C - M g + \frac{e P}{\sqrt{d^2 + e^2}} = 0$$

$$\left( \begin{array}{c} N_C \\ P \end{array} \right) = \text{Find}(N_C, P)$$

$$N_C = 280.2 \text{ N} \quad P = 140 \text{ N} \quad \text{Ans.}$$

$$\Sigma M_O = 0; \quad -\mu_s N_C \left( \frac{a}{2} \right) - N_I x + \left( \frac{e P}{\sqrt{d^2 + e^2}} \right) \left( \frac{b}{2} \right) = 0$$

$$x = \frac{-1}{2} \frac{\mu_s N_C a \sqrt{d^2 + e^2} - e P b}{N_C \sqrt{d^2 + e^2}} \quad x = 123.51 \text{ mm}$$

Thus, the distance from $A$ is 

$$A = x + \frac{b}{2} \quad A = 523.51 \text{ mm} \quad \text{Ans.}$$
Determine the friction force on the crate of mass $M$, and the resultant normal force and its position $x$, measured from point $A$, if the force is $P$.

**Given:**
- $M = 40$ kg
- $a = 400$ mm
- $b = 800$ mm
- $c = 200$ mm
- $P = 300$ N

**Solution:**

**Initial guesses:** $F_C = 25$ N $N_C = 100$ N

Given

$$
\Sigma F_x = 0; \quad P \left( \frac{d}{\sqrt{d^2 + e^2}} \right) - F_C = 0
$$

$$
\Sigma F_y = 0; \quad N_C - Mg + P \left( \frac{e}{\sqrt{d^2 + e^2}} \right) = 0
$$

$$
\begin{pmatrix} F_C \\ N_C \end{pmatrix} = \text{Find}(F_C, N_C) \quad F_{C_{\text{max}}} = \mu_s N_C
$$

Since $F_C = 180.00$ N > $F_{C_{\text{max}}} = 76.13$ N then the crate slips

$$
F_C = \mu_k N_C \quad \begin{pmatrix} F_C \\ N_C \end{pmatrix} = \begin{pmatrix} 30.5 \\ 152.3 \end{pmatrix} \text{ N}
$$

$$
\Sigma M_O = 0; \quad -N_C x + P \left( \frac{e}{\sqrt{d^2 + e^2}} \right) a - P \left( \frac{d}{\sqrt{d^2 + e^2}} \right) c = 0
$$

$$
x = -P \left( \frac{-ea + dc}{N_C \sqrt{d^2 + e^2}} \right)
$$

Since $x = 0.39$ m < $\frac{b}{2} = 0.40$ m

Then the block does not tip.

Ans. $x_f = a + x, \quad x_f = 0.79$ m
Determine the smallest force the man must exert on the rope in order to move the 80-kg crate. Also, what is the angle \( \theta \) at this moment? The coefficient of static friction between the crate and the floor is \( \mu_s = 0.3 \).

**SOLUTION**

Crate:

\[ \sum F_x = 0; \quad 0.3N_C - T' \sin \theta = 0 \]

\[ \sum F_y = 0; \quad N_C + T' \cos \theta - 80(9.81) = 0 \]

Pulley:

\[ \sum F_x = 0; \quad -T \cos 30^\circ + T \cos 45^\circ + T' \sin \theta = 0 \]

\[ \sum F_y = 0; \quad T \sin 30^\circ + T \sin 45^\circ - T' \cos \theta = 0 \]

Thus,

\[ T = 6.29253 \cdot T' \sin \theta \]

\[ T = 0.828427 \cdot T' \cos \theta \]

\[ \theta = \tan^{-1} \left( \frac{0.828427}{6.29253} \right) = 7.50^\circ \]

\[ T = 0.82134 \cdot T' \]

From Eqs. (1) and (2),

\[ N_C = 239 \text{ N} \]

\[ T' = 550 \text{ N} \]

So that

\[ T = 452 \text{ N} \]
8–40.

The spool of wire having a mass $M$ rests on the ground at $A$ and against the wall at $B$. Determine the force $P$ required to begin pulling the wire horizontally off the spool. The coefficient of static friction between the spool and its points of contact is $\mu_s$.

Units Used:

$$\text{kN} = 10^3 \text{ N}$$

Given:

$$M = 150 \text{ kg}$$
$$\mu_s = 0.25$$
$$a = 0.45 \text{ m}$$
$$b = 0.25 \text{ m}$$

Solution:

Initial guesses: $P = 100 \text{ N}$  $F_A = 10 \text{ N}$  $N_A = 20 \text{ N}$  $N_B = 30 \text{ N}$  $F_B = 10 \text{ N}$

Given

$$\Sigma F_y = 0; \quad N_A + F_B - Mg = 0$$
$$\Sigma F_x = 0; \quad F_A - N_B + P = 0$$
$$\Sigma M_B = 0; \quad -Pb + Mga - Na + Fa\ a = 0$$
$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$\begin{pmatrix} P \\ F_A \\ F_B \\ N_A \\ N_B \end{pmatrix} = \text{Find}(P, F_A, F_B, N_A, N_B)$$

$$\begin{pmatrix} F_A \\ N_A \\ F_B \\ N_B \end{pmatrix} = \begin{pmatrix} 0.28 \\ 1.12 \\ 0.36 \\ 1.42 \end{pmatrix} \text{ kN} \quad P = 1.14 \text{ kN} \quad \text{Ans.}$$
The spool of wire having a mass \( M \) rests on the ground at \( A \) and against the wall at \( B \). Determine the forces acting on the spool at \( A \) and \( B \) for the given force \( P \). The coefficient of static friction between the spool and the ground at point \( A \) is \( \mu_s \). The wall at \( B \) is smooth.

Units Used:

\[ \text{kN} = 10^3 \text{ N} \]

Given:

\[ P = 800 \text{ N} \quad a = 0.45 \text{ m} \]
\[ M = 150 \text{ kg} \quad b = 0.25 \text{ m} \]
\[ \mu_s = 0.35 \]

Solution: Assume no slipping

Initial guesses: \( F_A = 10 \text{ N} \quad N_A = 10 \text{ N} \quad N_B = 10 \text{ N} \quad F_{A\text{max}} = 10 \text{ N} \)

Given

\[ \Sigma F_x = 0; \quad F_A - N_B + P = 0 \]
\[ \Sigma F_y = 0; \quad N_A - Mg = 0 \]
\[ \Sigma M_O = 0; \quad -P b + F_A a = 0 \]
\[ F_{A\text{max}} = \mu_s N_A \]

\[
\begin{bmatrix}
F_A \\
F_{A\text{max}} \\
N_A \\
N_B
\end{bmatrix}
= \text{Find}(F_A, F_{A\text{max}}, N_A, N_B)
\]

\[
\begin{bmatrix}
F_A \\
F_{A\text{max}} \\
N_A \\
N_B
\end{bmatrix}
= \begin{bmatrix}
444 \\
515
\end{bmatrix} \text{ N}
\]

If \( F_A = 444 \text{ N} < F_{A\text{max}} = 515 \text{ N} \) then our no-slip assumption is good.

\[
\begin{bmatrix}
N_A \\
F_A
\end{bmatrix}
= \begin{bmatrix}
1.47 \\
0.44
\end{bmatrix} \text{ kN}
\]
\[
N_B = 1.24 \text{ kN}
\]
The friction hook is made from a fixed frame which is shown colored and a cylinder of negligible weight. A piece of paper is placed between the smooth wall and the cylinder. If \( \theta = 20^\circ \), determine the smallest coefficient of static friction \( \mu \) at all points of contact so that any weight \( W \) of paper \( p \) can be held.

**SOLUTION**

Paper:

\[ + \uparrow \Sigma F_y = 0; \quad F = 0.5W \]
\[ F = \mu N; \quad F = \mu N \]
\[ N = \frac{0.5W}{\mu} \]

Cylinder:

\[ \zeta + \Sigma M_O = 0; \quad F = 0.5W \]
\[ \downarrow \Sigma F_x = 0; \quad N \cos 20^\circ + F \sin 20^\circ - \frac{0.5W}{\mu} = 0 \]
\[ + \uparrow \Sigma F_y = 0; \quad N \sin 20^\circ - F \cos 20^\circ - 0.5W = 0 \]
\[ F = \mu N; \quad \mu^2 \sin 20^\circ + 2\mu \cos 20^\circ - \sin 20^\circ = 0 \]

\[ \mu = 0.176 \quad \text{Ans.} \]
The uniform rod has a mass of 10 kg and rests on the inside of the smooth ring at B and on the ground at A. If the rod is on the verge of slipping, determine the coefficient of static friction between the rod and the ground.

**SOLUTION**

\[\sum m_A = 0; \quad N_B (0.4) - 98.1 (0.25 \cos 30^\circ) = 0\]

\[N_B = 53.10 \text{ N}\]

\[\sum F_y = 0; \quad N_A - 98.1 + 53.10 \cos 30^\circ = 0\]

\[N_A = 52.12 \text{ N}\]

\[\sum F_x = 0; \quad \mu (52.12) - 53.10 \sin 30^\circ = 0\]

\[\mu = 0.509\]

Ans.
The rings \( A \) and \( C \) each weigh \( W \) and rest on the rod, which has a coefficient of static friction of \( \mu_s \). If the suspended ring at \( B \) has a weight of \( 2W \), determine the largest distance \( d \) between \( A \) and \( C \) so that no motion occurs. Neglect the weight of the wire. The wire is smooth and has a total length of \( l \).

**SOLUTION**

**Free-Body Diagram:** The tension developed in the wire can be obtained by considering the equilibrium of the free-body diagram shown in Fig. \( a \).

\[
+ \sum F_y = 0; \quad 2T \sin \theta - 2W = 0 \quad T = \frac{W}{\sin \theta}
\]

Due to the symmetrical loading and system, rings \( A \) and \( C \) will slip simultaneously. Thus, it's sufficient to consider the equilibrium of either ring. Here, the equilibrium of ring \( C \) will be considered. Since ring \( C \) is required to be on the verge of sliding to the left, the friction force \( F_C \) must act to the right such that \( F_C = \mu_s N_C \) as indicated on the free-body diagram of the ring shown in Fig. \( b \).

**Equations of Equilibrium:** Using the result of \( T \) and referring to Fig. \( b \), we have

\[
+ \sum F_y = 0; \quad N_C - w - \left[ \frac{W}{\sin \theta} \right] \sin \theta = 0 \quad N_C = 2w
\]

\[
\pm \sum F_x = 0; \quad \mu_s (2w) - \left[ \frac{W}{\sin \theta} \right] \cos \theta = 0 \quad \tan \theta = \frac{1}{2 \mu_s}
\]

From the geometry of Fig. \( c \), we find that \( \tan \theta = \frac{\sqrt{l^2 - \left( \frac{d}{2} \right)^2}}{\frac{d}{2}} = \frac{\sqrt{l^2 - d^2}}{d} \).

Thus,

\[
\frac{\sqrt{l^2 - d^2}}{d} = \frac{1}{2 \mu_s}
\]

\[
d = \frac{2 \mu_s l}{\sqrt{1 + 4 \mu_s^2}}
\]

**Ans.**
8-45. Car A has a mass of 1.4 Mg and mass center at G. If car B exerts a horizontal force on A of 2 kN, determine if this force is great enough to move car A. The coefficients of static and kinetic friction between the tires and the road are \( \mu_s = 0.5 \) and \( \mu_k = 0.35 \). Assume B's bumper is smooth.

Slipping:

\[ \vec{\Sigma} F = 0; \quad F - 2 = 0 \]
\[ F = 2 \text{ kN} \]
\[ + \vec{\Sigma} C = 0; \quad N_A = 13.734 \text{ kN} \]
\[ F_{net} = 0.5(13.734) = 6.867 \text{ kN} > 2 \text{ kN} \]

Tipping:

\[ \Sigma M_C = 0; \quad 2(0.5) < 13.734(0.8) \]
\[ 1 < 10.99 \]

Therefore car A will not move. Ans.
The beam $AB$ has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass of 50 kg and negligible thickness. Determine the minimum force $P$ needed to move the post. The coefficients of static friction at $B$ and $C$ are $\mu_B = 0.4$ and $\mu_C = 0.2$, respectively.

**SOLUTION**

**Member $AB$:**

$$\zeta + \Sigma M_A = 0; \quad -800 \left(\frac{4}{3}\right) + N_B (2) = 0$$

$$N_B = 533.3 \text{ N}$$

**Post:**

Assume slipping occurs at $C$; $F_C = 0.2 N_C$

$$\zeta + \Sigma M_C = 0; \quad -\frac{4}{5} P (0.3) + F_B (0.7) = 0$$

$$\downarrow \sum F_x = 0; \quad \frac{4}{5} P - F_B - 0.2 N_C = 0$$

$$\uparrow \sum F_y = 0; \quad \frac{3}{5} P + N_C - 533.3 - 50(9.81) = 0$$

$$P = 355 \text{ N}$$

$$N_C = 811.0 \text{ N}$$

$$F_B = 121.6 \text{ N}$$

$$(F_B)_{\text{max}} = 0.4(533.3) = 213.3 \text{ N} > 121.6 \text{ N} \quad \text{(O.K.)}$$
The beam $AB$ has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass of 50 kg and negligible thickness. Determine the two coefficients of static friction at $B$ and at $C$ so that when the magnitude of the applied force is increased to $P = 150$ N, the post slips at both $B$ and $C$ simultaneously.

**SOLUTION**

**Member $AB$:**

\[ \zeta + \sum M_A = 0; \quad -800 \left( \frac{4}{3} \right) + N_B (2) = 0 \]

\[ N_B = 533.3 \text{ N} \]

**Post:**

\[ \sum F_y = 0; \quad N_C - 533.3 + 150 \left( \frac{3}{5} \right) - 50(9.81) = 0 \]

\[ N_C = 933.83 \text{ N} \]

\[ \zeta + \sum M_C = 0; \quad -4 \left( 150 \right)(0.3) + F_B (0.7) = 0 \]

\[ F_B = 51.429 \text{ N} \]

\[ \sum F_x = 0; \quad \frac{4}{5} (150) - F_C - 51.429 = 0 \]

\[ F_C = 68.571 \text{ N} \]

\[ \mu_C = \frac{F_C}{N_C} = \frac{68.571}{933.83} = 0.0734 \]

\[ \mu_B = \frac{F_B}{N_B} = \frac{51.429}{533.3} = 0.0964 \]
The beam $AB$ has a negligible mass and thickness and is subjected to a force of 200 N. It is supported at one end by a pin and at the other end by a spool having a mass of 40 kg. If a cable is wrapped around the inner core of the spool, determine the minimum cable force $P$ needed to move the spool. The coefficients of static friction at $B$ and $D$ are $\mu_B = 0.4$ and $\mu_D = 0.2$, respectively.

**SOLUTION**

*Equations of Equilibrium:* From FBD (a),

$$\zeta + \sum M_A = 0; \quad N_B \ 3 - 200(2) = 0 \quad N_B = 133.33 \text{ N}$$

From FBD (b),

$$\sum F_y = 0; \quad P - F_B - F_D = 0 \quad (1)$$

$$\sum M_D = 0; \quad F_B (0.4) - P(0.2) = 0 \quad (2)$$

*Friction:* Assuming the spool slips at point $B$, then $F_B = \mu_B N_B = 0.4(133.33) = 53.33 \text{ N}$. Substituting this value into Eqs. (1) and (2) and solving, we have

$$F_D = 53.33 \text{ N} \quad P = 106.67 \text{ N} = 107 \text{ N} \quad \text{Ans.}$$

Since $(F_D)_{\text{max}} = \mu_D N_D = 0.2(525.73) = 105.15 \text{ N} > F_D$, the spool does not slip at point $D$. Therefore the above assumption is correct.
8.49. The 45-kg disk rests on the surface for which the coefficient of static friction is \( \mu_s = 0.2 \). Determine the largest couple moment \( M \) that can be applied to the bar without causing motion.

\[ \begin{align*}
\sum X &= 0; \quad F_x = F_y = 0.2N_k \\
\sum F_y &= 0; \quad B_x - 0.2N_k = 0 \\
\sum F_x &= 0; \quad N_a - B_y - 45(9.81) = 0
\end{align*} \]

- \( N_a = 551.8 \text{ N} \)
- \( B_x = 110.4 \text{ N} \)
- \( B_y = 110.4 \text{ N} \)

\( \sum M = 0; \quad -110.4(0.3) - 110.4(0.4) + M = 0 \)

\( M = 77.3 \text{ N}\cdot\text{m} \quad \text{Ans.} \)
8-50. The 45-kg disk rests on the surface for which the coefficient of static friction is $\mu_s = 0.15$. If $M = 50 \text{ N} \cdot \text{m}$, determine the friction force at $A$.

**Bar:**

$\Sigma \tau_{C} = 0$:

$-B_y(0.3) - B_x(0.4) + 50 = 0$

$\Sigma F_x = 0$:

$B_x = C_x$

$\Sigma F_y = 0$:

$B_y = C_y$

**Disk:**

$\Sigma F_x = 0$:

$B_x = F_x$

$\Sigma \tau_A = 0$:

$N_d - B_x - 45(9.81) = 0$

$\Sigma M_A = 0$:

$B_y(0.125) - F_x(0.125) = 0$

$N_d = 512.9 \text{ N}$

$F_x = 71.4 \text{ N}$

$F_x = 0.15(512.9) = 76.93 \text{ N} > 71.43 \text{ N}$

No motion of disk.
8–51.

The block of weight $W$ is being pulled up the inclined plane of slope $\alpha$ using a force $P$. If $P$ acts at the angle $\phi$ as shown, show that for slipping to occur, $P = W \sin(\alpha + \theta)/\cos(\phi - \theta)$, where $\theta$ is the angle of friction; $\theta = \tan^{-1} \mu$.

**SOLUTION**

\[ \sum F_x = 0; \quad P \cos \phi - W \sin \alpha - \mu N = 0 \]

\[ \sum F_y = 0; \quad N - W \cos \alpha + P \sin \phi = 0 \]

\[
P \cos \phi - W \sin \alpha - \mu (W \cos \alpha - P \sin \phi) = 0
\]

\[
P = W \left( \frac{\sin \alpha + \mu \cos \alpha}{\cos \phi + \mu \sin \phi} \right)
\]

Let $\mu = \tan \theta$

\[
P = W \left( \frac{\sin (\alpha + \theta)}{\cos (\phi - \theta)} \right)
\]

(QED)
Determine the angle φ at which P should act on the block so that the magnitude of P is as small as possible to begin pushing the block up the incline. What is the corresponding value of P? The block weighs W and the slope α is known.

**SOLUTION**

Slipping occurs when \( P = W \left( \frac{\sin(\alpha + \theta)}{\cos(\phi - \theta)} \right) \) where θ is the angle of friction \( \theta = \tan^{-1} \theta \).

\[
\frac{dP}{d\phi} = W \left( \frac{\sin(\alpha + \theta) \sin(\phi - \theta)}{\cos^2(\phi - \theta)} \right) = 0
\]

\( \sin(\alpha + \theta) \sin(\phi - \theta) = 0 \)

\( \sin(\alpha + \theta) = 0 \) or \( \sin(\phi - \theta) = 0 \)

\( \alpha = -\theta \quad \phi = \theta \quad \text{Ans.} \)

\( P = W \sin(\alpha + \phi) \quad \text{Ans.} \)
8.53. The homogeneous semicylinder has a mass \( m \) and mass center at \( G \). Determine the largest angle \( \theta \) of the inclined plane upon which it rests so that it does not slip down the plane. The coefficient of static friction between the plane and the cylinder is \( \mu_s = 0.3 \). Also, what is the angle \( \phi \) for this case?

The semi cylinder is a two-force member:

Since \( F = \mu N \)

\[
\tan \theta = \frac{\mu N}{N} = \mu
\]

\[
\theta = \tan^{-1} 0.3 = 16.7^\circ \quad \text{Ans}
\]

\[
\frac{r}{\sin(180^\circ - \phi)} = \frac{4}{\sin 16.7^\circ}
\]

\[
0.671 = \sin \phi
\]

\[
\phi = 42.6^\circ \quad \text{Ans}
\]
The uniform beam has a weight $W$ and length $4a$. It rests on the fixed rails at $A$ and $B$. If the coefficient of static friction at the rails is $\mu_s$, determine the horizontal force $P$, applied perpendicular to the face of the beam, which will cause the beam to move.

**SOLUTION**

From FBD (a),

$\uparrow \Sigma F = 0; \quad N_A + N_B - W = 0$

$\zeta + \Sigma M_B = 0; \quad -N_A(3a) + W(2a) = 0$

$$N_A = \frac{2}{3}W \quad N_B = \frac{1}{3}W$$

Support $A$ can sustain twice as much static frictional force as support $B$.

From FBD (b),

$\uparrow \Sigma F = 0; \quad P + F_B - F_A = 0$

$\zeta + \Sigma M_B = 0; \quad -P(4a) + F_A(3a) = 0$

$$F_A = \frac{4}{3}P \quad F_B = \frac{1}{3}P$$

The frictional load at $A$ is 4 times as great as at $B$. The beam will slip at $A$ first.

$$P = \frac{3}{4}(F_A)_{\text{max}} = \frac{3}{4}(\mu_s N_A) = \frac{1}{2}\mu_s W$$

Ans.
8–55.

Determine the greatest angle \( \theta \) so that the ladder does not slip when it supports the 75-kg man in the position shown. The surface is rather slippery, where the coefficient of static friction at \( A \) and \( B \) is \( \mu_s = 0.3 \).

**SOLUTION**

*Free-Body Diagram:* The slipping could occur at either end \( A \) or \( B \) of the ladder. We will assume that slipping occurs at end \( B \). Thus, \( F_B = \mu_s N_B = 0.3 N_B \).

*Equations of Equilibrium:* Referring to the free-body diagram shown in Fig. \( b \), we have

\[
\begin{align*}
\sum F_x &= 0; & F_{BC} \sin \theta/2 - 0.3 N_B &= 0 \\
& & F_{BC} \sin \theta/2 &= 0.3 N_B \\
\sum F_y &= 0; & N_B - F_{BC} \cos \theta/2 &= 0 \\
& & F_{BC} \cos \theta/2 &= N_B \quad (2)
\end{align*}
\]

Dividing Eq. (1) by Eq. (2) yields

\[ \tan \theta/2 = 0.3 \]

\[ \theta = 33.40^\circ = 33.4^\circ \quad \text{Ans.} \]

Using this result and referring to the free-body diagram of member \( AC \) shown in Fig. \( a \), we have

\[
\begin{align*}
\zeta + \sum M_A &= 0; & F_{BC} \sin 33.40^\circ (2.5) - 75 \times 9.81 \times 0.25 &= 0 \\
& & F_{BC} &= 133.66 \text{ N} \\
\sum F_x &= 0; & F_A - 133.66 \sin \left( \frac{33.40^\circ}{2} \right) &= 0 \\
& & F_A &= 38.40 \text{ N} \\
\sum F_y &= 0; & N_A + 133.66 \cos \left( \frac{33.40^\circ}{2} \right) - 75 \times 9.81 &= 0 \\
& & N_A &= 607.73 \text{ N}
\end{align*}
\]

Since \( F_A < (F_A)_{\max} = \mu_s N_A = 0.3 \times 607.73 = 182.32 \text{ N} \), end \( A \) will not slip. Thus, the above assumption is correct.
The uniform 6-kg slender rod rests on the top center of the 3-kg block. If the coefficients of static friction at the points of contact are \( \mu_A = 0.4, \mu_B = 0.6, \) and \( \mu_C = 0.3, \) determine the largest couple moment \( M \) which can be applied to the rod without causing motion of the rod.

**SOLUTION**

**Equations of Equilibrium:** From FBD (a),

\[
\begin{align*}
\sum F_x &= 0; \quad F_B - N_C = 0 \quad (1) \\
\sum F_y &= 0; \quad N_B + F_C - 58.86 = 0 \quad (2) \\
\sum M &= 0; \quad F_C(0.6) + N_C(0.8) - M - 58.86(0.3) = 0 \quad (3)
\end{align*}
\]

From FBD (b),

\[
\begin{align*}
\sum F_x &= 0; \quad F_A - F_B = 0 \quad (4) \\
\sum F_y &= 0; \quad N_A - N_B - 29.43 = 0 \quad (5) \\
\sum M &= 0; \quad F_B(0.3) - N_B(x) - 29.43(x) = 0 \quad (6)
\end{align*}
\]

**Friction:** Assume slipping occurs at point \( C \) and the block tips, then \( F_C = \mu_s C N_C = 0.3N_c \) and \( x = 0.1 \text{ m.} \) Substituting these values into Eqs. (1), (2), (3), (4), (5), and (6) and solving, we have

\[
M = 8.561 \text{ N} \cdot \text{m} = 8.56 \text{ N} \cdot \text{m}
\]

Ans.

\[
N_B = 50.83 \text{ N} \quad N_A = 80.26 \text{ N} \quad F_A = F_B = N_C = 26.75 \text{ N}
\]

Since \( (F_A)_{\text{max}} = \mu_s A N_A = 0.4(80.26) = 32.11 \text{ N} > F_A, \) the block does not slip. Also, \( (F_B)_{\text{max}} = \mu_s B N_B = 0.6(50.83) = 30.50 \text{ N} > F_B, \) then slipping does not occur at point \( B. \) Therefore, the above assumption is correct.
The disk has a weight $W$ and lies on a plane which has a coefficient of static friction $\mu$. Determine the maximum height $h$ to which the plane can be lifted without causing the disk to slip.

**SOLUTION**

**Unit Vector:** The unit vector perpendicular to the inclined plane can be determined using cross product.

$$\mathbf{A} = (0 - 0)i + (0 - a)j + (h - 0)k = -aj + h\mathbf{k}$$
$$\mathbf{B} = (2a - 0)i + (0 - a)j + (0 - 0)k = 2ai - aj$$

Then

$$\mathbf{N} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} i & j & k \\ 0 & -a & h \\ 2a & -a & 0 \end{vmatrix} = ahi + 2ahj + 2a^2k$$

$$n = \frac{\mathbf{N}}{N} = \frac{ahi + 2ahj + 2a^2k}{a\sqrt{5h^2 + 4a^2}}$$

Thus

$$\cos \gamma = \frac{2a}{\sqrt{5h^2 + 4a^2}} \quad \text{hence} \quad \sin \gamma = \frac{\sqrt{5}h}{\sqrt{5h^2 + 4a^2}}$$

**Equations of Equilibrium and Friction:** When the disk is on the verge of sliding down the plane, $F = \mu N$.

$$\Sigma F_n = 0; \quad N - W \cos \gamma = 0 \quad N = W \cos \gamma \quad (1)$$
$$\Sigma F_i = 0; \quad W \sin \gamma - \mu N = 0 \quad N = \frac{W \sin \gamma}{\mu} \quad (2)$$

Divide Eq. (2) by (1) yields

$$\frac{\sin \gamma}{\mu \cos \gamma} = 1$$
$$\frac{\sqrt{5}h}{\sqrt{5h^2 + 4a^2}} \frac{2a}{\mu \left(\frac{2a}{\sqrt{5h^2 + 4a^2}}\right)} = 1$$

$$h = \frac{2a \mu}{\sqrt{5}}$$

Ans.
Determine the largest angle \( \theta \) that will cause the wedge to be self-locking regardless of the magnitude of horizontal force \( P \) applied to the blocks. The coefficient of static friction between the wedge and the blocks is \( \mu_s = 0.3 \). Neglect the weight of the wedge.

**SOLUTION**

**Free-Body Diagram:** For the wedge to be self-locking, the frictional force \( F \) indicated on the free-body diagram of the wedge shown in Fig. a must act downward and its magnitude must be \( F \leq \mu_s N = 0.3N \).

**Equations of Equilibrium:** Referring to Fig. a, we have
\[
+ \uparrow \Sigma F_y = 0; \quad 2N \sin \theta/2 - 2F \cos \theta/2 = 0
\]
\[
F = N \tan \theta/2
\]

Using the requirement \( F \leq 0.3N \), we obtain
\[
N \tan \theta/2 \leq 0.3N
\]

\[
\theta = 33.4^\circ
\]

\( \text{Ans.} \)
If the beam $AD$ is loaded as shown, determine the horizontal force $P$ which must be applied to the wedge in order to remove it from under the beam. The coefficients of static friction at the wedge’s top and bottom surfaces are $\mu_{CA} = 0.25$ and $\mu_{CB} = 0.35$, respectively. If $P = 0$, is the wedge self-locking? Neglect the weight and size of the wedge and the thickness of the beam.

**SOLUTION**

*Equations of Equilibrium and Friction:* If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_A = \mu_{sA} N_A = 0.25 N_A$ and $F_B = \mu_{sB} N_B = 0.35 N_B$. From FBD (a),

$$\zeta + \Sigma M_D = 0; \quad N_A \cos 10^\circ(7) + 0.25N_A \sin 10^\circ(7) - 6.00(2) - 16.0(5) = 0$$

$$N_A = 12.78 \text{ kN}$$

From FBD (b),

$$\uparrow \Sigma F_y = 0; \quad N_B - 12.78 \sin 80^\circ - 0.25(12.78) \sin 10^\circ = 0$$

$$N_B = 13.14 \text{ kN}$$

$$\downarrow \Sigma F_x = 0; \quad P + 12.78 \cos 80^\circ - 0.25(12.78) \cos 10^\circ - 0.35(13.14) = 0$$

$$P = 5.53 \text{ kN}$$  \textbf{Ans.}

Since a force $P (> 0)$ is required to pull out the wedge, \textit{the wedge will be self-locking when $P = 0.$}  \textbf{Ans.}
The wedge has a negligible weight and a coefficient of static friction \( \mu_s = 0.35 \) with all contacting surfaces. Determine the largest angle \( \theta \) so that it is “self-locking.” This requires no slipping for any magnitude of the force \( P \) applied to the joint.

**SOLUTION**

**Friction:** When the wedge is on the verge of slipping, then \( F = \mu N = 0.35N \). From the force diagram (\( P \) is the ‘locking’ force.),

\[
\tan \frac{\theta}{2} = \frac{0.35N}{N} = 0.35
\]

\[\theta = 38.6^\circ\] Ans.
8–61.

If the spring is compressed 60 mm and the coefficient of static friction between the tapered stub S and the slider A is \( \mu_{SA} = 0.5 \), determine the horizontal force \( P \) needed to move the slider forward. The stub is free to move without friction within the fixed collar C. The coefficient of static friction between A and surface B is \( \mu_{AB} = 0.4 \). Neglect the weights of the slider and stub.

**SOLUTION**

Stub:

\[ + \Sigma F_y = 0; \quad N_A \cos 30^\circ - 0.5N_A \sin 30^\circ - 300(0.06) = 0 \]

\[ N_A = 29.22 \text{ N} \]

Slider:

\[ + \Sigma F_y = 0; \quad N_B - 29.22 \cos 30^\circ + 0.5(29.22) \sin 30^\circ = 0 \]

\[ N_B = 18 \text{ N} \]

\[ \downarrow \Sigma F_x = 0; \quad P - 0.4(18) - 29.22 \sin 30^\circ - 0.5(29.22) \cos 30^\circ = 0 \]

\[ P = 34.5 \text{ N} \quad \text{Ans.} \]
If \( P = 250 \) N, determine the required minimum compression in the spring so that the wedge will not move to the right. Neglect the weight of \( A \) and \( B \). The coefficient of static friction for all contacting surfaces is \( \mu_s = 0.35 \). Neglect friction at the rollers.

**SOLUTION**

*Free-Body Diagram:* The spring force acting on the cylinder is \( F_{sp} = kx = 15(10^3)x \). Since it is required that the wedge is on the verge to slide to the right, the frictional force must act to the left on the top and bottom surfaces of the wedge and their magnitude can be determined using friction formula.

\[
(F_f)_1 = \mu N_1 = 0.35N_1 \quad (F_f)_2 = 0.35N_2
\]

*Equations of Equilibrium:* Referring to the FBD of the cylinder, Fig. \( a \),

\[
+ \sum F_y = 0; \quad N_1 - 15(10^3)x = 0 \quad N_1 = 15(10^3)x
\]

Thus, \( (F_f)_1 = 0.35[15(10^3)x] = 5.25(10^3)x \)

Referring to the FBD of the wedge shown in Fig. \( b \),

\[
+ \sum F_y = 0; \quad N_2 \cos 10^\circ - 0.35N_2 \sin 10^\circ - 15(10^3)x = 0
\]

\[
N_2 = 16.233(10^3)x
\]

\[
\pm \sum F_x = 0; \quad 250 - 5.25(10^3)x - 0.35[16.233(10^3)x] \cos 10^\circ
\]

\[
- [16.233(10^3)x] \sin 10^\circ = 0
\]

\[
x = 0.01830 \text{ m} = 18.3 \text{ mm}
\]

Ans.
Determine the minimum applied force $P$ required to move wedge $A$ to the right. The spring is compressed a distance of 175 mm. Neglect the weight of $A$ and $B$. The coefficient of static friction for all contacting surfaces is $\mu_s = 0.35$. Neglect friction at the rollers.

**SOLUTION**

*Equations of Equilibrium and Friction:* Using the spring formula, $F_{sp} = kx = 15(0.175) = 2.625$ kN. If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_A = \mu_s N_A = 0.35 N_A$ and $F_B = \mu_s N_B = 0.35 N_B$. From FBD (a),

\[ P - 0.35(2.625) - 0.35(2.841) \cos 10^\circ - 2.841 \sin 10^\circ = 0 \]

\[ P = 2.39 \text{ kN} \]

From FBD (b),

\[ + \sum F_y = 0; \quad N_B - 2.625 = 0 \quad N_B = 2.625 \text{ kN} \]

\[ N_A \cos 10^\circ - 0.35 N_A \sin 10^\circ - 2.625 = 0 \]

\[ N_A = 2.841 \text{ kN} \]

\[ - \sum F_x = 0; \quad P - 0.35(2.625) - 0.35(2.841) \cos 10^\circ - 2.841 \sin 10^\circ = 0 \]

\[ P = 2.39 \text{ kN} \]
The wedge blocks are used to hold the specimen in a tension testing machine. Determine the design angle $\theta$ of the wedges so that the specimen will not slip regardless of the applied load. The coefficients of static friction are $\mu_A = 0.1$ at $A$ and $\mu_B = 0.6$ at $B$. Neglect the weight of the blocks.

**Specimen:**

\[ S_0 = \frac{P}{2} \]

**Wedge:**

\[ \sum F_x = 0: \quad N_A \cos \theta - 0.1N_A \sin \theta - \frac{P}{0.6} = 0 \]

\[ \sum F_y = 0: \quad 0.1N_A \cos \theta + N_A \sin \theta - \frac{P}{2} = 0 \]

\[ P = 2N_A(0.1 \cos \theta + \sin \theta) \]

\[ 0.6N_A(\cos \theta - 0.1 \sin \theta) - N_A(0.1 \cos \theta + \sin \theta) = 0 \]

\[ 0.5 \cos \theta - 1.06 \sin \theta = 0 \]

\[ \theta = \tan^{-1}\left(\frac{0.5}{1.06}\right) = 25.5^\circ \]

*Ans*
The coefficient of static friction between wedges $B$ and $C$ is $\mu_s = 0.6$ and between the surfaces of contact $B$ and $A$ and $C$ and $D$, $\mu_s = 0.4$. If the spring is compressed 200 mm when in the position shown, determine the smallest force $P$ needed to move wedge $C$ to the left. Neglect the weight of the wedges.

**SOLUTION**

Wedge $B$:

\[ \sum F_x = 0; \quad N_{AB} - 0.6N_{BC} \cos 15^\circ - N_{BC} \sin 15^\circ = 0 \]

\[ \sum F_y = 0; \quad N_{BC} \cos 15^\circ - 0.6N_{BC} \sin 15^\circ - 0.4N_{AB} - 100 = 0 \]

\[ N_{BC} = 210.4 \text{ N} \]

\[ N_{AB} = 176.4 \text{ N} \]

Wedge $C$:

\[ \sum F_y = 0; \quad N_{CD} \cos 15^\circ - 0.4N_{CD} \sin 15^\circ + 0.6(210.4) \sin 15^\circ - 210.4 \cos 15^\circ = 0 \]

\[ N_{CD} = 197.8 \text{ N} \]

\[ \sum F_x = 0; \quad 197.8 \sin 15^\circ + 0.4(197.8) \cos 15^\circ + 210.4 \sin 15^\circ + 0.6(210.4) \cos 15^\circ - P = 0 \]

\[ P = 304 \text{ N} \quad \text{Ans.} \]
The coefficient of static friction between the wedges $B$ and $C$ is $\mu_s = 0.6$ and between the surfaces of contact $B$ and $A$ and $C$ and $D$, $\mu_s' = 0.4$. If $P = 50$ N, determine the largest allowable compression of the spring without causing wedge $C$ to move to the left. Neglect the weight of the wedges.

**SOLUTION**

Wedge $C$:

\[ \sum F_x = 0; \quad (N_{CD} + N_{BC}) \sin 15^\circ + (0.4N_{CD} + 0.6N_{BC}) \cos 15^\circ - 50 = 0 \]

\[ \sum F_y = 0; \quad (N_{CD} - N_{BC}) \cos 15^\circ + (-0.4N_{CD} + 0.6N_{BC}) \sin 15^\circ = 0 \]

$N_{BC} = 34.61$ N

$N_{CD} = 32.53$ N

Wedge $B$:

\[ \sum F_x = 0; \quad N_{AB} - 0.6(34.61) \cos 15^\circ - 34.61 \sin 15^\circ = 0 \]

$N_{AB} = 29.01$ N

\[ \sum F_y = 0; \quad 34.61 \cos 15^\circ - 0.6(34.61) \sin 15^\circ - 0.4(29.01) - 500x = 0 \]

\[ x = 0.03290 \text{ m} = 32.9 \text{ mm} \quad \text{Ans.} \]
The vise is used to grip the pipe. If a horizontal force $F_1$ is applied perpendicular to the end of the handle of length $l$, determine the compressive force $F$ developed in the pipe. The square threads have a mean diameter $d$ and a lead $a$. How much force must be applied perpendicular to the handle to loosen the vise?

**Given:**

\[
F_1 := 100 \text{N} \\
d := 37.5 \text{mm} \\
\mu_s := 0.3 \\
L := 250 \text{mm} \\
a := 5 \text{mm}
\]

**Solution:**

\[
r := \frac{d}{2}
\]

\[
\theta := \arctan \left( \frac{a}{2 \cdot \pi \cdot r} \right) \\
\phi := \arctan (\mu_s) \\
\theta = 2.43 \text{ deg} \\
\phi = 16.70 \text{ deg}
\]

\[
F_1 \cdot L = F \cdot r \cdot \tan (\theta + \phi)
\]

\[
F := F_1 \cdot \frac{L}{r \cdot \tan (\theta + \phi)} \\
F = 3.84 \text{ kN} \quad \text{Ans.}
\]

To loosen screw,

\[
P \cdot L = F \cdot r \cdot \tan (\phi - \theta)
\]

\[
P := F \cdot r \cdot \frac{\tan (\phi - \theta)}{L} \\
P = 73.3 \text{ N} \quad \text{Ans.}
\]
Determine the couple forces $F$ that must be applied to the handle of the machinist’s vise in order to create a compressive force $F_A$ in the block. Neglect friction at the bearing $A$. The guide at $B$ is smooth so that the axial force on the screw is $F_A$. The single square-threaded screw has a mean radius $b$ and a lead $c$, and the coefficient of static friction is $\mu_s$.

**Given:**

\[
\begin{align*}
&\ a := 125\text{mm} \\
&\ F_A := 400\text{N} \\
&\ b := 6\text{mm} \\
&\ c := 8\text{mm} \\
&\ \mu_s := 0.27
\end{align*}
\]

**Solution:**

\[
\begin{align*}
&\ \phi := \tan^{-1}(\mu_s) \quad \phi = 15.11 \text{ deg} \\
&\ \theta := \tan^{-1}\left(\frac{c}{2\pi b}\right) \quad \theta = 11.981 \text{ deg} \quad \text{Ans.}
\end{align*}
\]

\[
F \cdot 2\cdot a = F_A \cdot b \cdot \tan(\theta + \phi)
\]

\[
F := F_A \cdot \frac{b}{2a} \cdot \tan(\theta + \phi) \quad F = 4.91 \text{ N} \quad \text{Ans.}
\]
The column is used to support the upper floor. If a force \( F = 80 \text{ N} \) is applied perpendicular to the handle to tighten the screw, determine the compressive force in the column. The square-threaded screw on the jack has a coefficient of static friction of \( \mu_s = 0.4 \), mean diameter of 25 mm, and a lead of 3 mm.

**SOLUTION**

\[
M = W(r) \tan(\phi_s + \theta_p)
\]

\[
\phi_s = \tan^{-1}(0.4) = 21.80^\circ
\]

\[
\theta_p = \tan^{-1}\left[\frac{3}{2\pi(12.5)}\right] = 2.188^\circ
\]

\[
80(0.5) = W(0.0125) \tan(21.80^\circ + 2.188^\circ)
\]

\[
W = 7.19 \text{ kN}
\]

*Ans.*
If the force $F$ is removed from the handle of the jack in Prob. 8-69, determine if the screw is self-locking.

**SOLUTION**

$\phi_s = \tan^{-1}(0.4) = 21.80^\circ$

$\theta_p = \tan^{-1}\left(\frac{3}{2\pi(12.5)}\right) = 2.188^\circ$

Since $\phi_s > \theta_p$, the screw is self-locking. 

Ans.
8–71.

If the clamping force at \( G \) is 900 N, determine the horizontal force \( F \) that must be applied perpendicular to the handle of the lever at \( E \). The mean diameter and lead of both single square-threaded screws at \( C \) and \( D \) are 25 mm and 5 mm, respectively. The coefficient of static friction is \( \mu_s = 0.3 \).

**SOLUTION**

Referring to the free-body diagram of member \( GAC \) shown in Fig. \( a \), we have

\[
\Sigma M_A = 0; F_{CD}(0.2) - 900(0.2) = 0 \quad F_{CD} = 900 \text{ N}
\]

Since the screw is being tightened, Eq. 8–3 should be used. Here,

\[
\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{5}{2\pi(12.5)}\right] = 3.643^\circ;
\]

\[
\phi_s = \tan^{-1} \mu_s = \tan^{-1}(0.3) = 16.699^\circ; \quad \text{and} \quad M = F(0.125). \quad \text{Since} \ M \text{ must overcome the friction of two screws,}
\]

\[
M = 2\left[W \tan(\phi_s + \theta)\right]
\]

\[
F(0.125) = 2 \left[900(0.0125)\tan(16.699^\circ + 3.643^\circ)\right]
\]

\[
F = 66.7 \text{ N}
\]

*Note: Since \( \phi_s > \theta \), the screw is self-locking.*
If a horizontal force of \( F = 50 \text{ N} \) is applied perpendicular to the handle of the lever at \( E \), determine the clamping force developed at \( G \). The mean diameter and lead of the single square-threaded screw at \( C \) and \( D \) are 25 mm and 5 mm, respectively. The coefficient of static friction is \( \mu_s = 0.3 \).

**SOLUTION**

Since the screw is being tightened, Eq. 8–3 should be used. Here, \( \theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left(\frac{5}{2\pi(12.5)}\right) = 3.643^\circ \);

\[ \phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699^\circ; \text{ and } M = 50(0.125). \] Since \( M \) must overcome the friction of two screws,

\[ M = 2[Wr \tan(\phi_s + \theta)] \]
\[ 50(0.125) = 2[F_{CD}(0.0125)\tan(16.699^\circ + 3.643^\circ)] \]
\[ F_{CD} = 674.32 \text{ N} \]

Using the result of \( F_{CD} \) and referring to the free-body diagram of member \( GAC \) shown in Fig. \( a \), we have

\[ \sum M_A = 0; 674.32(0.2) - F_G(0.2) = 0 \]

\[ F_G = 674 \text{ N} \]

**Note:** Since \( \phi_s > \theta \), the screws are self-locking.
A turnbuckle, similar to that shown in Fig. 8–17, is used to tension member AB of the truss. The coefficient of the static friction between the square threaded screws and the turnbuckle is \( \mu_s = 0.5 \). The screws have a mean radius of 6 mm and a lead of 3 mm. If a torque of \( M = 10 \text{ N} \cdot \text{m} \) is applied to the turnbuckle, to draw the screws closer together, determine the force in each member of the truss. No external forces act on the truss.

**SOLUTION**

**Frictional Forces on Screw:** Here, we must overcome friction. Applying Eq. 8–3, we have

\[
F_{AB} = 1380.62 \text{ N (T)} = 1.38 \text{ kN (T)}
\]

**Note:** Since \( \phi_s > \theta \), the screw is self-locking. It will not unscrew even if moment \( M \) is removed.

**Method of Joints:**

Joint B:

\[
\sum F_x = 0; \quad 1380.62 \left( \frac{3}{5} \right) - F_{BD} = 0
\]

\[ F_{BD} = 828.37 \text{ N (C)} = 828 \text{ N (C)} \quad \text{Ans.} \]

\[
\sum F_y = 0; \quad F_{BC} - 1380.62 \left( \frac{4}{5} \right) = 0
\]

\[ F_{BC} = 1104.50 \text{ N (C)} = 1.10 \text{ kN (C)} \quad \text{Ans.} \]

Joint A:

\[
\sum F_x = 0; \quad F_{AC} - 1380.62 \left( \frac{3}{5} \right) = 0
\]

\[ F_{AC} = 828.37 \text{ N (C)} = 828 \text{ N (C)} \quad \text{Ans.} \]

\[
\sum F_y = 0; \quad 1380.62 \left( \frac{4}{5} \right) - F_{AD} = 0
\]

\[ F_{AD} = 1104.50 \text{ N (C)} = 1.10 \text{ kN (C)} \quad \text{Ans.} \]

Joint C:

\[
\sum F_x = 0; \quad F_{CD} \left( \frac{3}{5} \right) - 828.37 = 0
\]

\[ F_{CD} = 1380.62 \text{ N (T)} = 1.38 \text{ kN (T)} \quad \text{Ans.} \]

\[
\sum F_y = 0; \quad C_y + 1380.62 \left( \frac{4}{5} \right) - 1104.50 = 0
\]

\[ C_y = 0 \quad \text{(No external applied load. check!)} \]
A turnbuckle, similar to that shown in Fig. 8–17, is used to tension member $AB$ of the truss. The coefficient of the static friction between the square-threaded screws and the turnbuckle is $\mu_s = 0.5$. The screws have a mean radius of 6 mm and a lead of 3 mm. Determine the torque $M$ which must be applied to the turnbuckle to draw the screws closer together, so that the compressive force of 500 N is developed in member $BC$.

**SOLUTION**

**Method of Joints:**

Joint $B$:

$$+ \sum F_y = 0; \quad 500 - F_{AB} \left( \frac{4}{5} \right) = 0 \quad F_{AB} = 625 \text{ N (C)}$$

**Frictional Forces on Screws:** Here, $\theta = \tan^{-1} \left( \frac{L}{2\pi r} \right) = \tan^{-1} \left( \frac{3}{2\pi(6)} \right) = 4.550^\circ$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.5) = 26.565^\circ$. Since friction at two screws must be overcome, then, $W = 2F_{AB} = 2(625) = 1250 \text{ N}$. Applying Eq. 8–3, we have

$$M = Wr \tan(\theta + \phi_s)$$

$$= 1250(0.006) \tan(4.550^\circ + 26.565^\circ)$$

$$= 4.53 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

**Note:** Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if moment $M$ is removed.
The shaft has a square-threaded screw with a lead of 8 mm and a mean radius of 15 mm. If it is in contact with a plate gear having a mean radius of 30 mm, determine the resisting torque \( M \) on the plate gear which can be overcome if a torque of 7 N·m is applied to the shaft. The coefficient of static friction at the screw is \( \mu_b = 0.2 \). Neglect friction of the bearings located at \( A \) and \( B \).

**SOLUTION**

**Frictional Forces on Screw:** Here, \( \theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left(\frac{8}{2\pi(15)}\right) = 4.852^\circ \), \( W = F, M = 7 \text{ N·m} \) and \( \phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.310^\circ \). Applying Eq. 8-3, we have

\[
M = W r \tan (\theta + \phi)
\]

\[
7 = F (0.015) \tan (4.852^\circ + 11.310^\circ)
\]

\[
F = 1610.29 \text{ N}
\]

**Note:** Since \( \phi_s > \theta \), the screw is self-locking. It will not unscrew even if force \( F \) is removed.

**Equations of Equilibrium:**

\[
\zeta + \sum M_O = 0; \quad 1610.29(0.03) - M = 0
\]

\[
M = 48.3 \text{ N·m}
\]

**Ans.**
8.76. The square threaded screw of the clamp has a mean diameter of 14 mm and a lead of 6 mm. If $\mu_s = 0.2$ for the threads, and the torque applied to the handle is $1.5 \text{ N} \cdot \text{m}$, determine the compressive force $F$ on the block.

*Frictional Forces on Screw:* Here, $\theta = \tan^{-1} \left( \frac{1}{2\pi r} \right) = \tan^{-1} \left( \frac{6}{2\pi(7)} \right) = 7.768^\circ$.

$W = F$ and $\phi_s = \tan^{-1} \mu_s = \tan^{-1} (0.2) = 11.310^\circ$. Applying Eq. 8-3, we have

$$M = W \tan (\theta + \phi)$$
$$1.5 = F(0.007) \tan (7.768^\circ + 11.310^\circ)$$

Ans

$F = 620 \text{ N}$

*Note:* Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if the moment is removed.
The fixture clamp consists of a square-threaded screw having a coefficient of static friction of \( \mu_s = 0.3 \), mean diameter of 3 mm, and a lead of 1 mm. The five points indicated are pin connections. Determine the clamping force at the smooth blocks D and E when a torque of \( M = 0.08 \text{ N} \cdot \text{m} \) is applied to the handle of the screw.

**SOLUTION**

**Frictional Forces on Screw:** Here, \( \theta = \tan^{-1} \left( \frac{l}{2\pi r} \right) = \tan^{-1} \left[ \frac{1}{2\pi (1.5)} \right] = 6.057^\circ \), \( W = P \), \( M = 0.08 \text{ N} \cdot \text{m} \) and \( \phi_s = \tan^{-1} \mu_s = \tan^{-1} (0.3) = 16.699^\circ \). Applying Eq. 8–3, we have

\[
M = Wr \tan(\theta + \phi_s)
\]
\[
0.08 = P(0.0015) \tan(6.057^\circ + 16.699^\circ)
\]
\[
P = 127.15 \text{ N}
\]

*Note:* Since \( \phi_s > \theta \), the screw is self-locking. It will not unscrew even if moment \( M \) is removed.

**Equation of Equilibrium:**

\[
\zeta + \Sigma M = 0; \quad 127.15 \cos 45^\circ (40) - F_E \cos 45^\circ (40) - F_E \sin 45^\circ (30) = 0
\]

\[
F_E = 72.66 \text{ N} = 72.7 \text{ N}
\]

Ans.

The equilibrium of the clamped blocks requires that

\[
F_D = F_E = 72.7 \text{ N}
\]

Ans.
The braking mechanism consists of two pinned arms and a square-threaded screw with left and right-hand threads. Thus when turned, the screw draws the two arms together. If the lead of the screw is 4 mm, the mean diameter 12 mm, and the coefficient of static friction is \( \mu_s = 0.35 \), determine the tension in the screw when a torque of 5 N m is applied to tighten the screw. If the coefficient of static friction between the brake pads \( A \) and \( B \) and the circular shaft is \( \mu_s = 0.5 \), determine the maximum torque \( M \) the brake can resist.

**SOLUTION**

**Frictional Forces on Screw:** Here, \( \theta = \tan^{-1}\left( \frac{1}{2\pi r} \right) = \tan^{-1}\left( \frac{4}{2\pi \cdot 6} \right) = 6.057^\circ \), \( M = 5 \text{ N} \cdot \text{m} \) and \( \phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.35) = 19.290^\circ \). Since friction at two screws must be overcome, then, \( W = 2P \). Applying Eq. 8–3, we have

\[
M = Wr \tan(\theta + \phi)
\]

\[
5 = 2P(0.006) \tan(6.057^\circ + 19.290^\circ)
\]

\[
P = 879.61 \text{ N} = 880 \text{ N}
\]

**Ans.**

**Note:** Since \( \phi_s > \theta \), the screw is self-locking. It will not unscrew even if moment \( M \) is removed.

**Equations of Equilibrium and Friction:** Since the shaft is on the verge to rotate about point \( O \), then, \( F_A = \mu_s N_A = 0.5N_A \) and \( F_B = \mu_s N_B = 0.5N_B \). From FBD (a),

\[
\sum M_D = 0; \quad 879.61(0.6) - N_B(0.3) = 0 \quad N_B = 1759.22 \text{ N}
\]

From FBD (b),

\[
\sum M_O = 0; \quad 2[0.5(1759.22)](0.2) - M = 0 \quad M = 352 \text{ N} \cdot \text{m}
\]

**Ans.**
If a horizontal force of $P = 100 \text{ N}$ is applied perpendicular to the handle of the lever at $A$, determine the compressive force $F$ exerted on the material. Each single square-threaded screw has a mean diameter of 25 mm and a lead of 7.5 mm. The coefficient of static friction at all contacting surfaces of the wedges is $\mu_s = 0.2$, and the coefficient of static friction at the screw is $\mu'_s = 0.15$.

**SOLUTION**

Since the screws are being tightened, Eq. 8–3 should be used. Here,

$$
\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{7.5}{2\pi(12.5)}\right] = 5.455^\circ;
$$

$$
\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.15) = 8.531^\circ; \quad M = 100(0.25) = 25 \text{ N} \cdot \text{m}; \quad \text{and} \quad W = T,
$$

where $T$ is the tension in the screw shank. Since $M$ must overcome the friction of two screws,

$$
M = 2[W r_s \tan(\phi_s + \theta)]
$$

$$
= 25 = 2[T(0.0125) \tan(8.531^\circ + 5.455^\circ)]
$$

$$
T = 4015.09 \text{ N} = 4.02 \text{ kN}
$$

Ans.

Referring to the free-body diagram of wedge $B$ shown in Fig. a using the result of $T$, we have

\[ \sum F_x = 0; \quad 4015.09 - 0.2N' - 0.2N \cos 15^\circ - N \sin 15^\circ = 0 \quad (1) \]

\[ + \sum F_y = 0; \quad N' + 0.2N \sin 15^\circ - N \cos 15^\circ = 0 \quad (2) \]

Solving,

$$
N = 6324.60 \text{ N} \quad N' = 5781.71 \text{ N}
$$

Using the result of $N$ and referring to the free-body diagram of wedge $C$ shown in Fig. b, we have

\[ + \sum F_y = 0; \quad 2(6324.60) \cos 15^\circ - 2[0.2(6324.60) \sin 15^\circ] - F = 0 \]

\[ F = 11563.42 \text{ N} = 11.6 \text{ kN} \quad \text{Ans.} \]
Determine the horizontal force $P$ that must be applied perpendicular to the handle of the lever at $A$ in order to develop a compressive force of 12 kN on the material. Each single square-threaded screw has a mean diameter of 25 mm and a lead of 7.5 mm. The coefficient of static friction at all contacting surfaces of the wedges is $\mu_s = 0.2$, and the coefficient of static friction at the screw is $\mu_s' = 0.15$.

**SOLUTION**

Referring to the free-body diagram of wedge $C$ shown in Fig. $a$, we have

$$+ \uparrow \sum F_y = 0; \quad 2N \cos 15^\circ - 2[0.2N \sin 15^\circ] - 12000 = 0$$

$$N = 6563.39 \text{ N}$$

Using the result of $N$ and referring to the free-body diagram of wedge $B$ shown in Fig. $b$, we have

$$+ \uparrow \sum F_y = 0; \quad N' - 6563.39 \cos 15^\circ + 0.2(6563.39) \sin 15^\circ = 0$$

$$N' = 6000 \text{ N}$$

$$\pm \sum F_x = 0; \quad T - 6563.39 \sin 15^\circ - 0.2(6563.39) \cos 15^\circ - 0.2(6000) = 0$$

$$T = 4166.68 \text{ N}$$

Since the screw is being tightened, Eq. 8–3 should be used. Here,

$$\theta = \tan^{-1} \left[ \frac{L}{2\pi r} \right] = \tan^{-1} \left[ \frac{7.5}{2\pi(12.5)} \right] = 5.455^\circ;$$

$$\phi_s = \tan^{-1} \mu_s = \tan^{-1}(0.15) = 8.531^\circ; \quad M = P(0.25); \quad \text{and} \quad W = T = 4166.68 \text{ N.}$$

Since $M$ must overcome the friction of two screws,

$$M = 2[Wr \tan (\phi_s + \theta)]$$

$$P(0.25) = 2[4166.68(0.0125) \tan (8.531^\circ + 5.455^\circ)]$$

$$P = 104 \text{ N}$$

\[ \text{Ans.} \]
Determine the clamping force on the board $A$ if the screw of the “C” clamp is tightened with a twist of $M = 8 \text{ N} \cdot \text{m}$. The single square-threaded screw has a mean radius of 10 mm, a lead of 3 mm, and the coefficient of static friction is $\mu_s = 0.35$.

**SOLUTION**

$\phi_s = \tan^{-1}(0.35) = 19.29^\circ$

$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(10)}\right] = 2.734^\circ$

$M = W(r) \tan(\phi_s + \theta_p)$

$8 = P(0.01) \tan(19.29^\circ + 2.734^\circ)$

$P = 1978 \text{ N} = 1.98 \text{ kN}$

Ans.
If the required clamping force at the board $A$ is to be $50$ N, determine the torque $M$ that must be applied to the handle of the “C” clamp to tighten it down. The single square-threaded screw has a mean radius of $10$ mm, a lead of $3$ mm, and the coefficient of static friction is $\mu_s = 0.35$.

**SOLUTION**

$\phi_s = \tan^{-1}(0.35) = 19.29^\circ$

$\theta_p = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{3}{2\pi(10)}\right] = 2.734^\circ$

$M = W(r) \tan (\phi_s + \theta_p)$

$= 50(0.01) \tan (19.29^\circ + 2.734^\circ) = 0.202 \text{ N} \cdot \text{m}$

Ans.
A cylinder having a mass of 250 kg is to be supported by the cord which wraps over the pipe. Determine the smallest vertical force $F$ needed to support the load if the cord passes (a) once over the pipe, $\beta = 180^\circ$, and (b) two times over the pipe, $\beta = 540^\circ$. Take $\mu_s = 0.2$.

**SOLUTION**

*Frictional Force on Flat Belt:* Here, $T_1 = F$ and $T_2 = 250(9.81) = 2452.5$ N. Applying Eq. 8–6, we have

a) If $\beta = 180^\circ = \pi$ rad

$$T_2 = T_1 e^{\mu \beta}$$

$$2452.5 = Fe^{0.2\pi}$$

$$F = 1308.38 \text{ N} = 1.13 \text{ kN} \quad \text{Ans.}$$

b) If $\beta = 540^\circ = 3\pi$ rad

$$T_2 = T_1 e^{\mu \beta}$$

$$2452.5 = Fe^{0.2(3\pi)}$$

$$F = 372.38 \text{ N} \approx 372 \text{ N} \quad \text{Ans.}$$
A cylinder having a mass of 250 kg is to be supported by the cord which wraps over the pipe. Determine the largest vertical force $F$ that can be applied to the cord without moving the cylinder. The cord passes (a) once over the pipe, $\beta = 180^\circ$, and (b) two times over the pipe, $\beta = 540^\circ$. Take $\mu_s = 0.2$.

**SOLUTION**

**Frictional Force on Flat Belt:** Here, $T_1 = 250(9.81) = 2452.5$ N and $T_2 = F$. Applying Eq. 8–6, we have

a) If $\beta = 180^\circ = \pi$ rad

\[ T_2 = T_1 e^{\mu \beta} \]
\[ F = 2452.5 e^{0.2 \pi} \]
\[ F = 4597.10 \text{ N} = 4.60 \text{ kN} \] \textbf{Ans.}

b) If $\beta = 540^\circ = 3 \pi$ rad

\[ T_2 = T_1 e^{\mu \beta} \]
\[ F = 2452.5 e^{0.2(3 \pi)} \]
\[ F = 16152.32 \text{ N} = 16.2 \text{ kN} \] \textbf{Ans.}
8-85.

The cord supporting the cylinder of mass $M$ passes around three pegs, $A$, $B$, $C$, where the coefficient of friction is $\mu_s$. Determine the range of values for the magnitude of the horizontal force $P$ for which the cylinder will not move up or down.

Given:

- $M = 6$ kg
- $\theta = 45$ deg
- $\mu_s = 0.2$
- $g = 9.81 \text{ m/s}^2$

Solution:

Total angle $\beta = \frac{5}{2} \pi - 4\theta$  $\beta = 270.00 \text{ deg}$

Forces $P_{min} = Mg e^{\mu_s \beta}$  $P_{max} = Mg e^{\mu_s \beta}$

Answer $P_{min} = 15.9 \text{ N} < P < P_{max} = 217.4 \text{ N}$  Ans.
A force of \( P = 25 \) N is just sufficient to prevent the 20-kg cylinder from descending. Determine the required force \( P \) to begin lifting the cylinder. The rope passes over a rough peg with two and half turns.

**SOLUTION**

The coefficient of static friction \( \mu_s \) between the rope and the peg when the cylinder is on the verge of descending requires \( T_2 = 20(9.81) \) N, \( T_1 = P = 25 \) N and \( \beta = 2.5(2\pi) = 5\pi \) rad. Thus,

\[
T_2 = T_1e^{\mu_s\beta} \\
20(9.81) = 25e^{\mu_s(5\pi)} \\
\ln(7.848) = 5\pi\mu_s \\
\mu_s = 0.1312
\]

In the case of the cylinder ascending \( T_2 = P \) and \( T_1 = 20(9.81) \) N. Using the result of \( \mu_s \), we can write

\[
T_2 = T_1e^{\mu_s\beta} \\
P = 20(9.81)e^{0.1312(5\pi)} \\
= 1539.78 \text{ N} \\
= 1.54 \text{ kN}
\]

Ans.
The 20-kg cylinder $A$ and 50-kg cylinder $B$ are connected together using a rope that passes around a rough peg two and a half turns. If the cylinders are on the verge of moving, determine the coefficient of static friction between the rope and the peg.

**SOLUTION**

In this case, $T_1 = 50(9.81) \text{N}$, $T_2 = 20(9.81) \text{N}$ and $\beta_1 = 2.5(2\pi) = 5\pi \text{ rad}$. Thus,

\[
T_1 = T_2 e^{\mu \beta_1} \\
50(9.81) = 20(9.81)e^{\mu(5\pi)} \\
\ln 2.5 = \mu(5\pi) \\
\mu_s = 0.0583
\]

Ans.
The uniform bar $AB$ is supported by a rope that passes over a frictionless pulley at $C$ and a fixed peg at $D$. If the coefficient of static friction between the rope and the peg is $\mu_D$, determine the smallest distance $x$ from the end of the bar at which a force $F$ may be placed and not cause the bar to move.

Given:

- $F = 20$ N
- $a = 1$ m
- $\mu_D = 0.3$

Solution:

Initial guesses:

- $T_A = 5$ N
- $T_B = 10$ N
- $x = 10$ m

Given

- $\Sigma M_A = 0$;
- $-Fx + T_B a = 0$
- $\Sigma F_y = 0$;
- $T_A + T_B - F = 0$

$T_A = T_B \left(\frac{\pi}{2}\right)$

$$\begin{pmatrix} T_A \\ T_B \\ x \end{pmatrix} = \text{Find}(T_A, T_B, x)$$

$x = 0.38$ m  \hspace{1cm} \text{Ans.}$
The truck, which has a mass of 3.4 Mg, is to be lowered down the slope by a rope that is wrapped around a tree. If the wheels are free to roll and the man at $A$ can resist a pull of 300 N, determine the minimum number of turns the rope should be wrapped around the tree to lower the truck at a constant speed. The coefficient of kinetic friction between the tree and rope is $\mu_k = 0.3$.

**SOLUTION**

\[ + \sum F_x = 0; \quad T_2 - 33,354 \sin 20^\circ = 0 \]

\[ T_2 = 11,407.7 \]

\[ T_2 = T_1 e^{\mu_k \beta} \]

\[ 11,407.7 = 300 e^{0.3 \beta} \]

\[ \beta = 12.1275 \text{ rad} \]

(Approx. 2 turns (695°))
8–90.

The smooth beam is being hoisted using a rope which is wrapped around the beam and passes through a ring at A as shown. If the end of the rope is subjected to a tension $T$ and the coefficient of static friction between the rope and ring is $\mu_s = 0.3$, determine the angle of $\theta$ for equilibrium.

SOLUTION

Equation of Equilibrium:

\[ \sum F_\theta = 0; \quad T - 2T' \cos \frac{\theta}{2} = 0 \quad T = 2T' \cos \frac{\theta}{2} \quad (1) \]

Frictional Force on Flat Belt: Here, $\beta = \frac{\theta}{2}$, $T_2 = T$ and $T_1 = T'$. Applying Eq. 8–6 $T_2 = T_1 e^{\mu \beta}$, we have

\[ T = T' e^{0.3(\theta/2)} = T' e^{0.15 \theta} \quad (2) \]

Substituting Eq. (1) into (2) yields

\[ 2T' \cos \frac{\theta}{2} = T' e^{0.15 \theta} \]

\[ e^{0.15 \theta} = 2 \cos \frac{\theta}{2} \]

Solving by trial and error

\[ \theta = 1.73104 \text{ rad} = 99.2^\circ \quad \text{Ans.} \]

The other solution, which starts with $T' = T e^{0.3(\theta/2)}$ based on cinching the ring tight, is $2.4326 \text{ rad} = 139^\circ$. Any angle from $99.2^\circ$ to $139^\circ$ is equilibrium.
8.91.

A cable is attached to the plate \( B \) of mass \( M_B \), passes over a fixed peg at \( C \), and is attached to the block at \( A \). Using the coefficients of static friction shown, determine the smallest mass of block \( A \) so that it will prevent sliding motion of \( B \) down the plane.

Given:

\[
M_B = 20 \text{ kg} \quad \mu_A = 0.2 \\
\theta = 30 \text{ deg} \quad \mu_B = 0.3 \\
g = 9.81 \frac{m}{s^2} \quad \mu_C = 0.3
\]

Solution:

Initial guesses: \( T_1 = 1 \text{ N} \quad T_2 = 1 \text{ N} \quad N_A = 1 \text{ N} \quad N_B = 1 \text{ N} \quad M_A = 1 \text{ kg} \)

Given

Block \( A \):

\[
\Sigma F_x = 0; \quad T_1 - \mu_A N_A - M_A g \sin(\theta) = 0 \\
\Sigma F_y = 0; \quad N_A - M_A g \cos(\theta) = 0
\]

Plate \( B \):

\[
\Sigma F_x = 0; \quad T_2 - M_B g \sin(\theta) + \mu_B N_B + \mu_A N_A = 0 \\
\Sigma F_y = 0; \quad N_B - N_A - M_B g \cos(\theta) = 0
\]

Peg \( C \):

\[
T_2 = T_1 e^{\mu_C \pi}
\]

Find \( (T_1, T_2, N_A, N_B, M_A) \)

\( M_A = 2.22 \text{ kg} \) \hspace{1cm} \text{Ans.}
8-92. Determine the smallest lever force $P$ needed to prevent the wheel from rotating if it is subjected to a torque of $M = 250 \text{ N} \cdot \text{m}$. The coefficient of static friction between the belt and the wheel is $\mu_s = 0.3$. The wheel is pin-connected at its center, $B$.

\[ \sum M_B = 0; \quad -F(200) + P(950) = 0 \]

\[ F = 4.75P \]

\[ \tau_1 = \tau_s \rho \]

\[ F' = 4.75P \cdot \rho \]

\[ F' = 19.53P \]

\[ \sum M_B = 0; \quad -19.53P(0.4) + 250 + 4.75P(0.4) = 0 \]

\[ P = 42.3 \text{ N} \quad \text{Ans} \]
8.93. Determine the torque $M$ that can be resisted by the band brake if a force of $P = 30$ N is applied to the handle of the lever. The coefficient of static friction between the belt and the wheel is $\mu_s = 0.3$. The wheel is pin-connected at its center, $B$.

\[ + \Sigma M_B = 0; \quad -F(200) + 30(350) = 0 \]

\[ F = 142.5 \text{ N} \]

\[ T_2 = T_1 e^{\alpha \beta} \]

\[ F' = 142.5 e^{-0.3(44)} = 585.8 \text{ N} \]

\[ -585.8(0.4) + 142.5(0.4) + M = 0 \]

$M = 177 \text{ N}\cdot\text{m}$
8.94.

Block $A$ has mass $m_A$ and rests on surface $B$ for which the coefficient of static friction is $\mu_{sAB}$. If the coefficient of static friction between the cord and the fixed peg at $C$ is $\mu_{sC}$, determine the greatest mass $m_D$ of the suspended cylinder $D$ without causing motion.

Given:

$m_A = 50 \text{ kg}$

$\mu_{sAB} = 0.25$

$\mu_{sC} = 0.3$

$a = 0.3 \text{ m}$

$b = 0.25 \text{ m}$

$c = 0.4 \text{ m}$

$d = 3$

$f = 4$

$g = 9.81 \frac{\text{ m}}{\text{s}^2}$

Solution: Assume block $A$ slips but does not tip. $\beta = \pi - \tan \left( \frac{f}{d} \right)$

The initial guesses: $N_B = 100 \text{ N}$ $T = 50 \text{ N}$ $m_D = 1 \text{ kg}$ $x = 10 \text{ mm}$

Given $m_{DG} = T e^{\mu_{sC} \beta}$

$\left( \frac{d}{\sqrt{f^2 + d^2}} \right) T - m_AG + N_B = 0$

$\left( \frac{-f}{\sqrt{f^2 + d^2}} \right) T + \mu_{sAB} N_B = 0$

$\left( \frac{f}{\sqrt{f^2 + d^2}} \right) Ta - \left( \frac{d}{\sqrt{f^2 + d^2}} \right) T \left( \frac{b}{2} \right) - N_B x = 0$

$\begin{bmatrix} N_B \\ T \\ m_D \\ x \end{bmatrix} = \text{Find} \left( N_B, T, m_D, x \right)$

$\left( \frac{N_B}{T} \right) = \begin{bmatrix} 413.05 \\ 129.08 \end{bmatrix}$  $N$  $m_D = 25.6 \text{ kg}$

$x = 0.052 \text{ m}$

Since $x = 51.6 \text{ mm} < \frac{b}{2} = 125 \text{ mm}$ our assumption is correct

$m_D = 25.6 \text{ kg}$  Ans.
8.95.

Block $A$ rests on the surface for which the coefficient of friction is $\mu_{sAB}$. If the mass of the suspended cylinder is $m_D$, determine the smallest mass $m_A$ of block $A$ so that it does not slip or tip. The coefficient of static friction between the cord and the fixed peg at $C$ is $\mu_{sC}$.

Units Used:

\begin{align*}
g &= 9.81 \text{ m/s}^2 \\
\text{Given:} \\
\mu_{sAB} &= 0.25 \\
m_D &= 4 \text{ kg} \\
\mu_{sC} &= 0.3 \\
a &= 0.3 \text{ m} \\
b &= 0.25 \text{ m} \\
c &= 0.4 \text{ m} \\
d &= 3 \\
f &= 4
\end{align*}

Solution: Assume that slipping is the critical motion $\beta = \pi - \arctan \left( \frac{f}{d} \right)$

The initial guesses: $N_B = 100 \text{ N}, T = 50 \text{ N}, m_A = 1 \text{ kg}, x = 10 \text{ mm}$

Given $m_Dg = Te^{\mu_{sC}\beta}$

\begin{align*}
\left( \frac{d}{\sqrt{f^2 + d^2}} \right)T - m_Ag + N_B &= 0 \\
\left( \frac{-f}{\sqrt{f^2 + d^2}} \right)T + \mu_{sAB}N_B &= 0 \\
\left( \frac{f}{\sqrt{f^2 + d^2}} \right)Ta - \left( \frac{d}{\sqrt{f^2 + d^2}} \right)T \frac{b}{2} - N_Bx &= 0
\end{align*}

\begin{align*}
\begin{bmatrix} N_B \\ T \\ m_A \\ x \end{bmatrix} = \text{Find}(N_B, T, m_A, x) \\
\begin{bmatrix} N_B \\ T \end{bmatrix} = \begin{bmatrix} 64.63 \\ 20.20 \end{bmatrix} \text{ N} \\
m_A = 7.82 \text{ kg} \\
x = 0.052 \text{ m}
\end{align*}

Since $x = 51.6 \text{ mm} < \frac{b}{2} = 125 \text{ mm}$ our assumption is correct $m_A = 7.82 \text{ kg}$ Ans.
8.96. The 20-kg motor has a center of gravity at G and is pin-connected at C to maintain a tension in the drive belt. Determine the smallest counterclockwise twist or torque $M$ that must be supplied by the motor to turn the disk B if wheel A locks and causes the belt to slip over the disk. No slipping occurs at A. The coefficient of static friction between the belt and the disk is $\mu_s = 0.3$.

**Equations of Equilibrium**: From FBD (a),

1. $\sum F_C = 0; \quad T_1 (100) + T_2 (200) - 196.2 (100) = 0 \quad [1]$

From FBD (b),

2. $\sum M_G = 0; \quad M + T_1 (0.05) - T_2 (0.05) = 0 \quad [2]$

**Frictional Force on Flat Belt**: Here, $\beta = 180^\circ = \pi$ rad. Applying Eq. 8-6, $T_2 = T_1 e^{\beta}$, we have

3. $T_1 = T_1 e^{0.3\pi} = 2.566T_1 \quad [3]$

Solving Eqs. [1], [2] and [3] yields

\[ M = 3.37 \text{ N} \cdot \text{m} \]

\[ T_1 = 42.97 \text{ N} \quad T_2 = 110.27 \text{ N} \]

Ans.
Determine the smallest force $P$ required to lift the 40-kg crate. The coefficient of static friction between the cable and each peg is $\mu_s = 0.1$.

**SOLUTION**

Since the crate is on the verge of ascending, $T_1 = 40(9.81)\,\text{N}$ and $T_2 = P$. From the geometry shown in Figs. $a$ and $b$, the total angle the rope makes when in contact with the peg is $\beta = 2\beta_1 + \beta_2 = 2\left(\frac{135^\circ}{180^\circ}\pi \right) + \left(\frac{90^\circ}{180^\circ}\pi \right) = 2\pi\,\text{rad}$. Thus,

$T_2 = T_1 e^{\mu_s}$

$P = 40(9.81)e^{0.1(2\pi)}$

$= 736\,\text{N}$

Ans.
Show that the frictional relationship between the belt tensions, the coefficient of friction $\mu$, and the angular contacts $\alpha$ and $\beta$ for the V-belt is $T_2 = T_1 e^{\mu \theta / \sin(\alpha / 2)}$.

**SOLUTION**

FBD of a section of the belt is shown.

Proceeding in the general manner:

\[ \Sigma F_x = 0; \quad -(T + dT) \cos \frac{\theta}{2} + T \cos \frac{\theta}{2} + 2 dF = 0 \]

\[ \Sigma F_y = 0; \quad -(T + dT) \sin \frac{\theta}{2} - T \sin \frac{\theta}{2} + 2 dN \sin \frac{\alpha}{2} = 0 \]

Replace

\[ \sin \frac{\theta}{2} \text{ by } \frac{\theta}{2}, \]

\[ \cos \frac{\theta}{2} \text{ by } 1, \]

\[ dF = \mu dN \]

Using this and $(dT)(d\theta) \to 0$, the above relations become

\[ dT = 2\mu dN \]

\[ T \ d\theta = 2 \left( dN \sin \frac{\alpha}{2} \right) \]

Combine

\[ \frac{dT}{T} = \mu \frac{d\theta}{\sin \frac{\alpha}{2}} \]

Integrate from $\theta = 0, T = T_1$

to $\theta = \beta, T = T_2$

we get,

\[ T_2 = T_1 e^{\mu \beta / \sin(\alpha / 2)} \]

Q.E.D
If a force of \( P = 200 \text{N} \) is applied to the handle of the bell crank, determine the maximum torque \( M \) that can be resisted so that the flywheel does not rotate clockwise. The coefficient of static friction between the brake band and the rim of the wheel is \( \mu_s = 0.3 \).

**SOLUTION**

Referring to the free-body diagram of the bell crane shown in Fig. \( a \) and the flywheel shown in Fig. \( b \), we have

\[
\begin{align*}
\zeta + \Sigma M_B &= 0; \quad T_A(0.3) + T_C(0.1) - 200(1) = 0 \quad (1) \\
\zeta + \Sigma M_O &= 0; \quad T_A(0.4) - T_C(0.4) - M = 0 \quad (2)
\end{align*}
\]

By considering the friction between the brake band and the rim of the wheel where
\[
\beta = \frac{270^\circ}{180^\circ} \pi = 1.5 \pi \text{ rad} \quad \text{and} \quad T_A > T_C,
\]
we can write

\[
\begin{align*}
T_A &= T_C e^{\mu_s \beta} \\
T_A &= T_C e^{0.3(1.5 \pi)} \\
T_A &= 4.1112 T_C \quad (3)
\end{align*}
\]

Solving Eqs. (1), (2), and (3) yields

\[
M = 187 \text{ N} \cdot \text{m}
\]

\[
T_A = 616.67 \text{ N} \quad T_C = 150.00 \text{ N} \quad \text{Ans.}
\]
A 10-kg cylinder $D$, which is attached to a small pulley $B$, is placed on the cord as shown. Determine the largest angle $\theta$ so that the cord does not slip over the peg at $C$. The cylinder at $E$ also has a mass of 10 kg, and the coefficient of static friction between the cord and the peg is $\mu_s = 0.1$.

**SOLUTION**

Since pulley $B$ is smooth, the tension in the cord between pegs $A$ and $C$ remains constant. Referring to the free-body diagram of the joint $B$ shown in Fig. $a$, we have

$$2T \sin \theta - 10(9.81) = 0 \quad T = \frac{49.05}{\sin \theta}$$

In the case where cylinder $E$ is on the verge of ascending, $T_2 = T = \frac{49.05}{\sin \theta}$ and $T_1 = 10(9.81) \text{ N}$. Here, $\frac{\pi}{2} + \theta$, Fig. $b$. Thus,

$$T_2 = T_1 e^{\mu_s \beta}$$
$$\frac{49.05}{\sin \theta} = 10(9.81) e^{0.1 \left( \frac{\pi}{2} + \theta \right)}$$
$$\ln \frac{0.5}{\sin \theta} = 0.1 \left( \frac{\pi}{2} + \theta \right)$$

Solving by trial and error, yields

$$\theta = 0.4221 \text{ rad} = 24.2^\circ$$

In the case where cylinder $E$ is on the verge of descending, $T_2 = 10(9.81) \text{ N}$ and $T_1 = \frac{49.05}{\sin \theta}$. Here, $\frac{\pi}{2} + \theta$. Thus,

$$T_2 = T_1 e^{\mu_s \beta}$$
$$10(9.81) = \frac{49.05}{\sin \theta} e^{0.1 \left( \frac{\pi}{2} + \theta \right)}$$
$$\ln (2 \sin \theta) = 0.1 \left( \frac{\pi}{2} + \theta \right)$$

Solving by trial and error, yields

$$\theta = 0.6764 \text{ rad} = 38.8^\circ$$

Thus, the range of $\theta$ at which the wire does not slip over peg $C$ is

$$24.2^\circ < \theta < 38.8^\circ$$

$$\theta_{\text{max}} = 38.8^\circ$$
A V-belt is used to connect the hub $A$ of the motor to wheel $B$. If the belt can withstand a maximum tension of $1200 \text{ N}$, determine the largest mass of cylinder $C$ that can be lifted and the corresponding torque $M$ that must be supplied to $A$. The coefficient of static friction between the hub and the belt is $\mu_s = 0.3$, and between the wheel and the belt is $\mu_s' = 0.20$. 

*Hint:* See Prob. 8–98.

**SOLUTION**

In this case, the maximum tension in the belt is $T_2 = 1200 \text{ N}$. Referring to the free-body diagram of hub $A$, shown in Fig. $a$, and the wheel $B$ shown in Fig. $b$, we have

\[
\sum M_D = 0; \quad M + T_1(0.15) - 1200(0.15) = 0
\]

\[
M = 0.15(1200 - T_1) \quad \text{(1)}
\]

\[
\sum M_D' = 0; \quad 1200(0.3) - T_1(0.3) - M_C(9.81)(0.2) = 0
\]

\[
1200 - T_1 = 6.54M_C \quad \text{(2)}
\]

If hub $A$ is on the verge of slipping, then

\[
T_2 = T_1 e^{\mu_s \beta_1 / \sin(\alpha/2)} \quad \text{where} \quad \beta_1 = \left( \frac{90^\circ + 75^\circ}{180^\circ} \right) \pi = 0.9167\pi \text{ rad}
\]

\[
T_1 = 213.19 \text{ N}
\]

Substituting $T_1 = 213.19 \text{ N}$ into Eq. (2), yields

\[
M_C = 150.89 \text{ kg}
\]

If wheel $B$ is on the verge of slipping, then

\[
T_2 = T_1 e^{\mu_s' \beta_1' / \sin(\alpha/2)} \quad \text{where} \quad \beta_2 = \left( \frac{180^\circ + 15^\circ}{180^\circ} \right) \pi = 1.0833\pi \text{ rad}
\]

\[
T_1 = 307.57 \text{ N}
\]

Substituting $T_1 = 307.57 \text{ N}$ into Eq. (2), yields

\[
M_C = 136.45 \text{ kg} = 136 \text{ kg (controls!)} \quad \text{Ans.}
\]

Substituting $T_1 = 307.57 \text{ N}$ into Eq. (1), we obtain

\[
M = 0.15(1200 - 307.57) = 134 \text{ N} \cdot \text{m} \quad \text{Ans.}
\]
The 20-kg motor has a center of gravity at \( G \) and is pin-connected at \( C \) to maintain a tension in the drive belt. Determine the smallest counterclockwise twist or torque \( M \) that must be supplied by the motor to turn the disk \( B \) if wheel \( A \) locks and causes the belt to slip over the disk. No slipping occurs at \( A \). The coefficient of static friction between the belt and the disk is \( \mu_s = 0.35 \).

**SOLUTION**

*Equations of Equilibrium:* From FBD (a),

\[ \zeta + \sum M_C = 0; \quad T_2(100) + T_1(200) - 196.2(100) = 0 \]  
(1)

From FBD (b),

\[ \zeta + \sum M_O = 0; \quad M + T_1(0.05) - T_2(0.05) = 0 \]  
(2)

*Frictional Force on Flat Belt:* Here, \( \beta = 180^\circ = \pi \text{ rad} \). Applying Eq. 8–6, \( T_2 = T_1 e^{\mu \beta} \), we have

\[ T_2 = T_1 e^{0.3\pi} = 3.003T_1 \]  
(3)

Solving Eqs. (1), (2), and (3) yields

\[ M = 3.93 \text{ N} \cdot \text{m} \]

\[ T_1 = 39.22 \text{ N} \quad T_2 = 117.8 \text{ N} \]

*Ans.*
Blocks $A$ and $B$ have a mass of 100 kg and 150 kg, respectively. If the coefficient of static friction between $A$ and $B$ and between $B$ and $C$ is $\mu_s = 0.25$ and between the ropes and the pegs $D$ and $E$, $\mu'_s = 0.5$ determine the smallest force $F$ needed to cause motion of block $B$ if $P = 30$ N.

**SOLUTION**

Assume no slipping between $A$ and $B$.

Peg $D$:

$$T_2 = T_1 e^{\mu_s}; \quad F_{AD} = 30 e^{0.5(45)} = 65.80 \text{ N}$$

Block $B$:

$$\sum F_x = 0; \quad -65.80 - 0.25 N_{BC} + F_{BE} \cos 45^\circ = 0$$

$$\sum F_y = 0; \quad N_{BC} - 981 + F_{BE} \sin 45^\circ - 150 (9.81) = 0$$

$$F_{BE} = 768.1 \text{ N}$$

$$N_{BC} = 1909.4 \text{ N}$$

Peg $E$:

$$T_2 = T_1 e^{\mu_s}; \quad F = 768.1 e^{0.5(45)} = 2.49 \text{ kN}$$

**Note:** Since $B$ moves to the right,

$$(F_{AB})_{\text{max}} = 0.25 (981) = 245.25 \text{ N}$$

$$245.25 = P_{\text{max}} e^{0.5(45)}$$

$$P_{\text{max}} = 112 \text{ N} > 30 \text{ N}$$

Hence, no slipping occurs between $A$ and $B$ as originally assumed.
Determine the minimum coefficient of static friction $\mu_s$ between the cable and the peg and the placement $d$ of the 3-kN force for the uniform 100-kg beam to maintain equilibrium.

**SOLUTION**

Referring to the free-body diagram of the beam shown in Fig. a, we have

\[ \sum F_x = 0; \quad T_{AB} \cos 45^\circ - T_{BC} \cos 60^\circ = 0 \]

\[ \sum F_y = 0; \quad T_{AB} \sin 45^\circ + T_{BC} \sin 60^\circ - 3 - \frac{100(9.81)}{1000} = 0 \]

\[ \sum M_A = 0; \quad T_{BC} \sin 60^\circ (6) - \frac{100(9.81)}{1000} (3) - 3d = 0 \]

Solving,

\[ d = 4.07 \text{ m} \]

\[ T_{BC} = 2.914 \text{ kN} \quad T_{AB} = 2.061 \text{ kN} \]

Using the results for $T_{BC}$ and $T_{AB}$ and considering the friction between the cable and the peg, where $\beta = \left( \frac{45^\circ + 60^\circ}{180^\circ} \pi \right) = 0.5833 \pi$ rad, we have

\[ T_{BC} = T_{AB} e^{\mu_s \beta} \]

\[ 2.914 = 2.061 e^{\mu_s (0.5833 \pi)} \]

\[ \ln 1.414 = \mu_s (0.5833 \pi) \]

\[ \mu_s = 0.189 \]

Ans.
A conveyer belt is used to transfer granular material and the frictional resistance on the top of the belt is \( F = 500 \, \text{N} \). Determine the smallest stretch of the spring attached to the moveable axle of the idle pulley \( B \) so that the belt does not slip at the drive pulley \( A \) when the torque \( M \) is applied. What minimum torque \( M \) is required to keep the belt moving? The coefficient of static friction between the belt and the wheel at \( A \) is \( \mu_s = 0.2 \).

**SOLUTION**

**Frictional Force on Flat Belt:** Here, \( \beta = 180^\circ = \pi \, \text{rad} \) and \( T_2 = 500 + T \) and \( T_1 = T \). Applying Eq. 8–6, we have

\[
T_2 = T_1 e^{\mu_s \beta} \\
500 + T = T e^{0.2 \pi} \\
T = 571.78 \, \text{N}
\]

**Equations of Equilibrium:** From FBD (a),

\[
\zeta + \sum M_O = 0; \quad M + 571.78(0.1) - (500 + 578.1)(0.1) = 0 \\
\text{Ans.} \quad M = 50.0 \, \text{N} \cdot \text{m}
\]

From FBD (b),

\[
\sum F_x = 0; \quad F_{sp} - 2(578.71) = 0 \quad F_{sp} = 1143.57 \, \text{N}
\]

Thus, the spring stretch is

\[
x = \frac{F_{sp}}{k} = \frac{1143.57}{4000} = 0.2859 \, \text{m} = 286 \, \text{mm}
\]

**Ans.**
The belt on the portable dryer wraps around the drum \( D \), idler pulley \( A \), and motor pulley \( B \). If the motor can develop a maximum torque of \( M = 0.80 \text{ N} \cdot \text{m} \), determine the smallest spring tension required to hold the belt from slipping. The coefficient of static friction between the belt and the drum and motor pulley is \( \mu_s = 0.3 \). Ignore the size of the idler pulley \( A \).

**SOLUTION**

\[
\zeta + \sum M_B = 0; \quad -T_1 (0.02) + T_2 (0.02) - 0.8 = 0
\]

\[
T_2 = T_1 e^{\mu_s}; \quad T_2 = T_1 e^{0.3(\pi)} = 2.5663T_1
\]

\[T_1 = 25.537 \text{ N}\]

\[T_2 = 65.53 \text{ N}\]

\[
\zeta + \sum M_C = 0; \quad -F_s (0.05) + (25.537 + 25.537 \sin 30^\circ)(0.1 \cos 45^\circ) + 25.537 \cos 30^\circ(0.1 \sin 45^\circ) = 0
\]

\[F_s = 85.4 \text{ N}\]
8-107. The collar bearing uniformly supports an axial force of $P = 2000$ N. If the coefficient of static friction is $\mu_s = 0.3$, determine the torque $M$ required to overcome friction.

**Bearing Friction:** Applying Eq. 8–7 with $R_2 = 37.5$ mm, $R_1 = 25$ mm, $\mu_s = 0.3$ and $P = 2000$ N, we have

$$M = \frac{2}{3} \mu_s P \left( \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \right)$$

$$= \frac{2}{3} \times 0.3 \times 2000 \times \left( \frac{37.5^3 - 25^3}{37.5^2 - 25^2} \right)$$

$$= 19000 \text{ N} \cdot \text{mm} = 19 \text{ N} \cdot \text{m} \quad \text{Ans.}$$
8-108. The collar bearing uniformly supports an axial force of $P = 2000$ N. If a torque of $M = 3.6$ N·m is applied to the shaft and causes it to rotate at constant velocity, determine the coefficient of kinetic friction at the surface of contact.

**Bearing Friction:** Applying Eq. 8-7 with $R_1 = 37.5$ mm, $R_2 = 25$ mm, $M = 3.6 \times (1000) = 3600$ N·mm and $P = 2000$ N, we have

$$M = \frac{2}{3} \mu_k P \left( \frac{R_1^3 - R_2^3}{R_2^3 - R_1^3} \right)$$

$$3600 = \frac{2}{3} (\mu_k)(2000) \left( \frac{37.5^3 - 25^3}{37.5^3 - 25^3} \right)$$

$\mu_k = 0.0568$ Ans.
The annular ring bearing is subjected to a thrust of 4000 N. If \( \mu_s = 0.35 \), determine the torque \( M \) that must be applied to overcome friction.

\[
M = \frac{2}{3} \mu_s P \left( \frac{R_2^2 - R_1^2}{R_2^2 - R_1^2} \right)
\]

\[
= \frac{2}{3} (0.35)(4000) \left( \frac{50^3 - 25^3}{50^3 - 25^3} \right)
\]

\[
= 54444 \text{ N} \cdot \text{mm}
\]

\( M = 54.4 \text{ N} \cdot \text{m} \)  

Ans.
The shaft is supported by a thrust bearing $A$ and a journal bearing $B$. Determine the torque $M$ required to rotate the shaft at constant angular velocity. The coefficient of kinetic friction at the thrust bearing is $\mu_k = 0.2$. Neglect friction at $B$.

**SOLUTION**

Applying Eq. 8–7 with $R_1 = \frac{0.075}{2} = 0.0375$ m, $R_2 = \frac{0.15}{2} = 0.075$ m, $\mu_k = 0.2$ and $P = 4000$ N, we have

\[
M = \frac{2}{3} \mu_k P \left( \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \right)
\]

\[
= \frac{2}{3} (0.2)(4000) \left( \frac{0.075^3 - 0.0375^3}{0.075^2 - 0.0375^2} \right)
\]

\[
= 46.7 \text{ N} \cdot \text{m}
\]

Ans.
The thrust bearing supports an axial load of \( P = 6 \text{ kN} \). If a torque of \( M = 150 \text{ N} \cdot \text{m} \) is required to rotate the shaft, determine the coefficient of static friction at the constant surface.

**SOLUTION**

Applying Eq. 8–7 with 
\[
R_1 = \frac{0.1 \text{ m}}{2} = 0.05 \text{ m}, \quad R_2 = \frac{0.2 \text{ m}}{2} = 0.1 \text{ m}, \quad M = 150 \text{ N} \cdot \text{m}
\]
and \( P = 6000 \text{ N} \), we have

\[
M = \frac{2}{3} \mu_s P \left( \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \right)
\]

\[
150 = \frac{2}{3} \mu_s (6000) \left( \frac{0.1^3 - 0.05^3}{0.1^2 - 0.05^2} \right)
\]

\[
\mu_s = 0.321 \quad \text{Ans.}
\]
Assuming that the variation of pressure at the bottom of the pivot bearing is defined as \( p = p_0 (R_2/r) \), determine the torque \( M \) needed to overcome friction if the shaft is subjected to an axial force \( P \). The coefficient of static friction is \( \mu_s \). For the solution, it is necessary to determine \( p_0 \) in terms of \( P \) and the bearing dimensions \( R_1 \) and \( R_2 \).

**SOLUTION**

\[ \Sigma F_z = 0; \quad P = \int_A dN = \int_0^{2\pi} \int_{R_1}^{R_2} pr \, dr \, d\theta \]

\[ = \int_0^{2\pi} \int_{R_1}^{R_2} p_0 \left( \frac{R_2}{r} \right) r \, dr \, d\theta \]

\[ = 2\pi p_0 R_2 (R_2 - R_1) \]

Thus, \( p_0 = \frac{P}{2\pi R_2 (R_2 - R_1)} \)

\[ \Sigma M_z = 0; \quad M = \int_A r \, dF = \int_0^{2\pi} \int_{R_1}^{R_2} \mu_s pr^2 \, dr \, d\theta \]

\[ = \int_0^{2\pi} \int_{R_1}^{R_2} \mu_s p_0 \left( \frac{R_2}{r} \right) r^2 \, dr \, d\theta \]

\[ = \mu_s (2\pi p_0) R_2 \frac{1}{2} (R_2^2 - R_1^2) \]

Using Eq. (1):

\[ M = \frac{1}{2} \mu_s P (R_2 + R_1) \]

**Ans.**
8–113.

The plate clutch consists of a flat plate $A$ that slides over the rotating shaft $S$. The shaft is fixed to the driving plate gear $B$. If the gear $C$, which is in mesh with $B$, is subjected to a torque of $M = 0.8 \text{N} \cdot \text{m}$, determine the smallest force $P$, that must be applied via the control arm, to stop the rotation. The coefficient of static friction between the plates $A$ and $D$ is $\mu_s = 0.4$. Assume the bearing pressure between $A$ and $D$ to be uniform.

**SOLUTION**

$$F = \frac{0.8}{0.03} = 26.667 \text{ N}$$

$$M = 26.667(0.150) = 4.00 \text{ N} \cdot \text{m}$$

$$M = \frac{2}{3} \mu P \left( \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \right)$$

$$4.00 = \frac{2}{3} (0.4) (P) \left( \frac{(0.125)^3 - (0.1)^3}{(0.125)^2 - (0.1)^2} \right)$$

$$P' = 88.525 \text{ N}$$

$$\zeta + \Sigma M_P = 0; \quad 88.525(0.2) - P(0.15) = 0$$

$$P = 118 \text{ N}$$
The conical bearing is subjected to a constant pressure distribution at its surface of contact. If the coefficient of static friction is \( \mu_s \), determine the torque \( M \) required to overcome friction if the shaft supports an axial force \( P \).

**SOLUTION**

The differential area (shaded) \( dA = 2\pi r \left( \frac{dr}{\cos \theta} \right) = \frac{2\pi rdr}{\cos \theta} \)

\[
P = \int p \cos \theta \, dA = \int p \cos \theta \left( \frac{2\pi rdr}{\cos \theta} \right) = 2\pi p \int_0^R rdr
\]

\[
P = \pi p R^2 \quad p = \frac{P}{\pi R^2}
\]

\[
dN = pdA = \frac{P}{\pi R^2} \left( \frac{2\pi rdr}{\cos \theta} \right) = \frac{2P}{R^2 \cos \theta} rdr
\]

\[
M = \int rdF = \int \mu_s r dN = \frac{2\mu_s P}{R^2 \cos \theta} \int_0^R r^2 dr
\]

\[
= \frac{2\mu_s P}{R^2 \cos \theta} \frac{R^3}{3} = \frac{2\mu_s P R}{3 \cos \theta}
\]

Ans.
The pivot bearing is subjected to a pressure distribution at its surface of contact which varies as shown. If the coefficient of static friction is μ, determine the torque $M$ required to overcome friction if the shaft supports an axial force $P$.

**SOLUTION**

$$dF = \mu \ dN = \mu \ \rho_0 \cos \left( \frac{\pi r}{2R} \right) \ dA$$

$$M = \int_A \mu \ \rho_0 \cos \left( \frac{\pi r}{2R} \right) \ r \ dr \ d\theta$$

$$= \mu \ \rho_0 \left[ \int_0^R \left( r^2 \cos \left( \frac{\pi r}{2R} \right) \right) \ dr \right] \int_0^{2\pi} \ d\theta$$

$$= \mu \ \rho_0 \left[ \frac{2r}{\left( \frac{\pi r}{2R} \right)^2} \cos \left( \frac{\pi r}{2R} \right) + \left( \frac{\pi r}{2R} \right)^2 - \frac{2}{\left( \frac{\pi r}{2R} \right)^3} \sin \left( \frac{\pi r}{2R} \right) \right]_0^R (2\pi)$$

$$= \mu \rho_0 \left[ \frac{16R^3}{\pi^3} \left( \frac{\pi}{2} \right)^2 - 2 \right]$$

$$= 0.7577 \mu \ \rho_0 \ \pi^3 R^3$$

$$P = \int_A \ dN = \int_0^R \rho_0 \cos \left( \frac{\pi r}{2R} \right) \ r \ dr \int_0^{2\pi} \ d\theta$$

$$= \rho_0 \left[ \frac{1}{\left( \frac{\pi r}{2R} \right)^2} \cos \left( \frac{\pi r}{2R} \right) + \frac{r}{\left( \frac{\pi r}{2R} \right) \sin \left( \frac{\pi r}{2R} \right)} \right]_0^R (2\pi)$$

$$= 4\rho_0 R^2 \left( 1 - \frac{2}{\pi} \right)$$

$$= 1.454 \rho_0 R^2$$

Thus, $M = 0.521 \ P \mu R$  

Ans.
A 200-mm diameter post is driven 3 m into sand for which \( \mu_s = 0.3 \). If the normal pressure acting \textit{completely around the post} varies linearly with depth as shown, determine the frictional torque \( M \) that must be overcome to rotate the post.

**SOLUTION**

\textit{Equations of Equilibrium and Friction:} The resultant normal force on the post is
\[
N = \frac{1}{2}(600 + 0)(3)(\pi)(0.2) = 180\pi \text{ N.}
\]
Since the post is on the verge of rotating,
\[
F = \mu_s N = 0.3(180\pi) = 54.0\pi \text{ N.}
\]
\[
\sum F = 0; \quad M - 54.0\pi(0.1) = 0
\]
\[M = 17.0 \text{ N} \cdot \text{m} \quad \text{Ans.}\]
A beam having a uniform weight $W$ rests on the rough horizontal surface having a coefficient of static friction $\mu_s$. If the horizontal force $P$ is applied perpendicular to the beam's length, determine the location $d$ of the point $O$ about which the beam begins to rotate.

**SOLUTION**

\[
w = \frac{\mu_s N}{L}
\]

\[
\Sigma F_z = 0; \quad N = W
\]

\[
\Sigma F_x = 0; \quad P + \frac{\mu_s N d}{L} - \frac{\mu_s N (L - d)}{L} = 0
\]

\[
\Sigma M_O = 0; \quad \frac{\mu_s N (L - d)^2}{2L} + \frac{\mu_s N d^2}{2L} - P \left( \frac{2L}{3} - d \right) = 0
\]

\[
\frac{\mu_s W (L - d)^2}{2L} + \frac{\mu_s W d^2}{2L} - \left( \frac{2L}{3} - d \right) \left( \frac{\mu_s W (L - d)}{L} - \frac{\mu_s W d}{L} \right) = 0
\]

\[3(L - d)^2 + 3d^2 - 2(2L - 3d)(L - 2d) = 0\]

\[6d^2 - 8Ld + L^2 = 0\]

Choose the root $d < L$.

\[d = 0.140 \, L\]  

Ans.
8-118. The connecting rod is attached to the piston by a 20-mm-diameter pin at \( B \) and to the crank shaft by a 50-mm-diameter bearing \( A \). If the piston is moving downwards, and the coefficient of static friction at these points is \( \mu_s = 0.2 \), determine the radius of the friction circle at each connection.

\[
\begin{align*}
(r_A)_B &= r_A \mu_s = \frac{50(0.2)}{2} = 5 \text{mm} \\
(r_B)_A &= r_B \mu_s = \frac{20(0.2)}{2} = 2 \text{mm}
\end{align*}
\]
8-119. The connecting rod is attached to the piston by a 20-mm-diameter pin at $B$ and to the crank shaft by a 50-mm-diameter bearing $A$. If the piston is moving upwards, and the coefficient of static friction at these points is $\mu_s = 0.3$, determine the radius of the friction circle at each connection.

\[
(r_l)_A = r_A \mu_s = 25 (0.3) = 7.50 \text{ mm}
\]

\[
(r_l)_B = r_B \mu_s = 10 (0.3) = 3 \text{ mm}
\]
8-120.

A pulley of mass $M$ has radius $a$ and the axle has a diameter $D$. If the coefficient of kinetic friction between the axle and the pulley is $\mu_k$ determine the vertical force $P$ on the rope required to lift the block of mass $M_B$ at constant velocity.

Given:

- $a = 120$ mm
- $M = 5$ kg
- $D = 40$ mm
- $\mu_k = 0.15$
- $M_B = 80$ kg

Solution:

\[
\phi_k = \tan(\mu_k)
\]

\[
r_f = \left(\frac{D}{2}\right) \sin(\phi_k)
\]

\[
\sum M_p = 0; \\
M_B g (a + r_f) + M g r_f - P (a - r_f) = 0
\]

\[
P = \frac{M_B g (a + r_f) + M g r_f}{a - r_f}
\]

Ans. $P = 826$ N
8-121.

A pulley of mass $M$ has radius $a$ and the axle has a diameter $D$. If the coefficient of kinetic friction between the axle and the pulley is $\mu_k$ determine the force $P$ on the rope required to lift the block of mass $M_B$ at constant velocity. Apply the force $P$ horizontally to the right (not as shown in the figure).

Given:

- $a = 120$ mm
- $M = 5$ kg
- $D = 40$ mm
- $\mu_k = 0.15$
- $M_B = 80$ kg

Solution:

\[ g = 9.81 \text{ m/s}^2 \]

\[ \phi_k = \tan(\mu_k) \]

\[ r_f = \frac{D}{2} \sin(\phi_k) \]

Guesses

\[ P = 1 \text{ N} \quad R = 1 \text{ N} \quad \alpha = 1 \text{ deg} \]

Given

\[ R \cos(\alpha) - M_B g - M g = 0 \]

\[ P - R \sin(\alpha) = 0 \]

\[ M_B g a - P a + R r_f = 0 \]

\[
\begin{pmatrix}
P \\
R \\
\alpha
\end{pmatrix} = \text{Find}(P, R, \alpha)
\]

\[ P = 814 \text{ N} \quad \text{Ans.} \]
The collar fits loosely around a fixed shaft that has radius $r$. If the coefficient of kinetic friction between the shaft and the collar is $\mu_k$, determine the force $P$ on the horizontal segment of the belt so that the collar rotates counterclockwise with a constant angular velocity. Assume that the belt does not slip on the collar; rather, the collar slips on the shaft. Neglect the weight and thickness of the belt and collar. The radius, measured from the center of the collar to the mean thickness of the belt is $R$.

Given:

- $r := 50\text{mm}$
- $\mu_k := 0.3$
- $R := 56.25\text{mm}$
- $F := 100\text{N}$

Solution:

- $\phi_k := \arctan(\mu_k)$
- $\phi_k = 16.699 \text{deg}$
- $r_f := r \sin(\phi_k)$
- $r_f = 14.3674 \text{mm}$

Equilibrium:

- $\sum F_y = 0; \quad R_y - F = 0 \quad R_y := F \quad R_y = 100.00 \text{N}$
- $\sum F_x = 0; \quad P - R_x = 0 \quad R_x = P$

Guess $P := 1\text{N}$

Given $-\sqrt{P^2 + F^2 \cdot r_f} + F \cdot R - P \cdot R = 0$ \quad $P := \text{Find}(P) \quad P = 68.97 \text{N}$ \quad Ans.
The collar fits loosely around a fixed shaft that has radius $r$. If the coefficient of kinetic friction between the shaft and the collar is $\mu_k$, determine the force $P$ on the horizontal segment of the belt so that the collar rotates clockwise with a constant angular velocity. Assume that the belt does not slip on the collar; rather, the collar slips on the shaft. Neglect the weight and thickness of the belt and collar. The radius, measured from the center of the collar to the mean thickness of the belt is $R$.

Given:

$r := 50\text{mm}$
$\mu_k := 0.3$
$R := 56.25\text{mm}$
$F := 100\text{N}$

Solution:

$\phi_k := \tan^{-1}(\mu_k)$
$\phi_k = 16.699 \text{deg}$

$r_f := r \cdot \sin(\phi_k)$
$r_f = 14.367 \text{mm}$

Equilibrium:

$\Sigma F_y = 0$: $R_y - F = 0$  $R_y := F$  $R_y = 100.00 \text{N}$

$\Sigma F_x = 0$: $P - R_x = 0$  $R_x = P$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{P^2 + F^2}$

Guess $P := 1\text{N}$

Given $\sqrt{P^2 + F^2} - r_f + F \cdot R - P \cdot R = 0$  $P := \text{Find}(P)$  $P = 145.0 \text{N}$  Ans.
A pulley having a diameter of 80 mm and mass of 1.25 kg is supported loosely on a shaft having a diameter of 20 mm. Determine the torque $M$ that must be applied to the pulley to cause it to rotate with constant motion. The coefficient of kinetic friction between the shaft and pulley is $\mu_k = 0.4$. Also calculate the angle $\theta$ which the normal force at the point of contact makes with the horizontal. The shaft itself cannot rotate.

**SOLUTION**

*Frictional Force on Journal Bearing:* Here, $\phi_k = \tan^{-1}\mu_k = \tan^{-1}0.4 = 21.80^\circ$. Then the radius of friction circle is $r_f = r \sin \phi_k = 0.01 \sin 21.80^\circ = 3.714 \times 10^{-3}$ m. The angle which the normal force makes with horizontal is

$$\theta = 90^\circ - \phi_k = 68.2^\circ$$

**Equations of Equilibrium:**

$$+ \sum F_y = 0; \quad R - 12.2625 = 0 \quad R = 12.2625 \text{ N}$$

$$\zeta + \sum M_O = 0; \quad 12.2625(3.714) \times 10^{-3} - M = 0$$

$$M = 0.0455 \text{ N} \cdot \text{m}$$
The 5-kg skateboard rolls down the 5° slope at constant speed. If the coefficient of kinetic friction between the 12.5 mm diameter axles and the wheels is \( \mu_k = 0.3 \), determine the radius of the wheels. Neglect rolling resistance of the wheels on the surface. The center of mass for the skateboard is at \( G \).

**SOLUTION**

Referring to the free-body diagram of the skateboard shown in Fig. a, we have

\[
\begin{align*}
\Sigma F_x &= 0; \\ F_x - 5(9.81) \sin 5^\circ &= 0 \\ F_x &= 4.275 \text{ N}
\end{align*}
\]

\[
\begin{align*}
\Sigma F_y &= 0; \\ N - 5(9.81) \cos 5^\circ &= 0 \\ N &= 48.86 \text{ N}
\end{align*}
\]

The effect of the forces acting on the wheels can be represented as if these forces are acting on a single wheel as indicated on the free-body diagram shown in Fig. b. We have

\[
\begin{align*}
\Sigma F_x &= 0; \\ R_x - 4.275 &= 0 \\ R_x &= 4.275 \text{ N}
\end{align*}
\]

\[
\begin{align*}
\Sigma F_y &= 0; \\ 48.86 - R_y &= 0 \\ R_y &= 48.86 \text{ N}
\end{align*}
\]

Thus, the magnitude of \( \mathbf{R} \) is

\[
R = \sqrt{R_x^2 + R_y^2} = \sqrt{4.275^2 + 48.86^2} = 49.05 \text{ N}
\]

\( \phi = \tan^{-1} \mu_s = \tan^{-1}(0.3) = 16.699^\circ \). Thus, the moment arm of \( \mathbf{R} \) from point \( O \) is \( (6.25 \sin 16.699^\circ) \) mm. Using these results and writing the moment equation about point \( O \), Fig. b, we have

\[
\zeta + \Sigma M_O = 0; \\ 4.275(r) - 49.05(6.25 \sin 16.699^\circ) = 0
\]

\( r = 20.6 \text{ mm} \)
8–126. Determine the force $P$ required to overcome rolling resistance and pull the 50-kg roller up the inclined plane with constant velocity. The coefficient of rolling resistance is $a = 15$ mm.

From the geometry indicated on the free-body diagram of the roller shown in Fig. a, $\theta = \sin^{-1}\left(\frac{15}{300}\right) = 2.866^\circ$.

We have

\[ \Sigma F_x = 0; \quad P \cos 30^\circ - 50(9.81) \sin 30^\circ - R \sin 2.866^\circ = 0 \]
\[ \Sigma F_y = 0; \quad P \sin 30^\circ + R \cos 2.866^\circ - 50(9.81) \cos 30^\circ = 0 \]

Solving,

$P = 299$ N

$R = 275.58$ N

$P$ can also be obtained directly by writing the moment equation of equilibrium about point $A$. Referring to Fig. a,

$\Sigma M_A = 0;$

$50(9.81) \sin(30^\circ + 2.866^\circ)(300) - P \cos(30^\circ - 2.866^\circ)(300) = 0$

$P = 299$ N
8–127. Determine the force $P$ required to overcome rolling resistance and support the 50-kg roller if it rolls down the inclined plane with constant velocity. The coefficient of rolling resistance is $a = 15$ mm.

From the geometry indicated on the free-body diagram of the roller shown in Fig. a, $\theta = \sin^{-1}\left(\frac{15}{300}\right) = 2.866^\circ$.

\[ \Sigma F_x = 0; \quad P \cos 30^\circ + R \sin 2.866^\circ - 50(9.81) \sin 30^\circ = 0 \]
\[ \Sigma F_y = 0; \quad P \sin 30^\circ + R \cos 2.866^\circ - 50(9.81) \cos 30^\circ = 0 \]

Solving,

\[ P = 266 \text{ N} \]
\[ R = 291.98 \text{ N} \]

$P$ can also be obtained directly by writing the moment equation of equilibrium about point A. Referring to Fig. a,

\[ \Sigma M_A = 0; \quad 50(9.81) \sin 30^\circ - 2.866^\circ \times (300) - P \cos 30^\circ + 2.866^\circ \times (300) = 0 \]

\[ P = 266 \text{ N} \]
The lawn roller has mass $M$. If the arm $BA$ is held at angle $\theta$ from the horizontal and the coefficient of rolling resistance for the roller is $r$, determine the force $P$ needed to push roller at constant speed. Neglect friction developed at the axle, $A$, and assume that the resultant force $P$ acting on the handle is applied along arm $BA$.

Given:

- $M = 80$ kg
- $\theta = 30$ deg
- $a = 250$ mm
- $r = 25$ mm

Solution:

\[
\theta_1 = \arcsin \left( \frac{r}{a} \right)
\]

\[
\theta_1 = 5.74 \text{ deg}
\]

\[
\Sigma M_0 = 0;
\]

\[
-rMg - P \sin(\theta)r + P \cos(\theta)a \cos(\theta_1) = 0
\]

\[
P = \frac{rMg}{-\sin(\theta) + r \cos(\theta) a \cos(\theta_1)}
\]

$P = 96.7$ N  \hspace{1cm} \text{Ans.}$
A machine of mass \( M \) is to be moved over a level surface using a series of rollers for which the coefficient of rolling resistance is \( a_g \) at the ground and \( a_m \) at the bottom surface of the machine. Determine the appropriate diameter of the rollers so that the machine can be pushed forward with a horizontal force \( P \). Hint: Use the result of Prob. 8-131.

Units Used:

\[
Mg = 1000 \text{ kg}
\]

Given:

\[
M = 1.4 \text{ Mg}
\]
\[
a_g = 0.5 \text{ mm}
\]
\[
a_m = 0.2 \text{ mm}
\]
\[
P = 250 \text{ N}
\]

Solution:

\[
\begin{align*}
P &= \frac{M g (a_g + a_m)}{2 r} \\
r &= Mg \left( \frac{a_g + a_m}{2P} \right) \quad r = 19.2 \text{ mm} \quad d = 2r \quad d = 38.5 \text{ mm} \quad \text{Ans.}
\end{align*}
\]
The hand cart has wheels with a diameter of 80 mm. If a crate having a mass of 500 kg is placed on the cart so that each wheel carries an equal load, determine the horizontal force $P$ that must be applied to the handle to overcome the rolling resistance. The coefficient of rolling resistance is 2 mm. Neglect the mass of the cart.

**SOLUTION**

\[
P \approx \frac{Wa}{r}
\]

\[
= 500(9.81)\left(\frac{2}{40}\right)
\]

\[
P = 245 \text{ N}
\]

Ans.
The cylinder is subjected to a load that has a weight $W$. If the coefficients of rolling resistance for the cylinder’s top and bottom surfaces are $a_A$ and $a_B$, respectively, show that a horizontal force having a magnitude of $P = [W(a_A + a_B)]/2r$ is required to move the load and thereby roll the cylinder forward. Neglect the weight of the cylinder.

**SOLUTION**

\[ \begin{align*}
\Sigma F_x &= 0; \quad (R_A)_x - P = 0 \quad (R_A)_x = P \\
\Sigma F_y &= 0; \quad (R_A)_y - W = 0 \quad (R_A)_y = W \\
\Sigma M_B &= 0; \quad P(r \cos \phi_A + r \cos \phi_B) - W(a_A + a_B) = 0 \quad (1)
\end{align*} \]

Since $\phi_A$ and $\phi_B$ are very small, $\cos \phi_A - \cos \phi_B = 1$. Hence, from Eq. (1)

\[ P = \frac{W(a_A + a_B)}{2r} \quad (QED) \]
A large crate having a mass of 200 kg is moved along the floor using a series of 150-mm-diameter rollers for which the coefficient of rolling resistance is 3 mm at the ground and 7 mm at the bottom surface of the crate. Determine the horizontal force $P$ needed to push the crate forward at a constant speed. *Hint:* Use the result of Prob. 8–131.

**SOLUTION**

**Rolling Resistance:** Applying the result obtained in Prob. 8–131. $P = \frac{W(a_A + a_B)}{2r}$, with $a_A = 7$ mm, $a_B = 3$ mm, $W = 200(9.81) = 1962$ N, and $r = 75$ mm, we have

\[
P = \frac{1962(7 + 3)}{2(75)} = 130.8 \text{ N} = 131 \text{ N}
\]

*Ans.*
Each of the cylinders has a mass of 50 kg. If the coefficients of static friction at the points of contact are \( \mu_A = 0.5, \mu_B = 0.5, \mu_C = 0.5, \) and \( \mu_D = 0.6, \) determine the smallest couple moment \( M \) needed to rotate cylinder \( E. \)

**Equations of Equilibrium:** From FBD (a).

\[
\begin{align*}
\sum F_y &= 0; \quad N_D - F_C = 0 \quad \text{[1]} \\
\sum \Sigma F_y &= 0; \quad N_C + F_B - 490.5 = 0 \quad \text{[2]} \\
\sum \Sigma M_D &= 0; \quad M - F_C (0.3) - F_D (0.3) = 0 \quad \text{[3]}
\end{align*}
\]

From FBD (b).

\[
\begin{align*}
\sum F_x &= 0; \quad N_A + F_B - N_D = 0 \quad \text{[4]} \\
\sum \Sigma F_y &= 0; \quad N_B - F_A - F_D = 490.5 = 0 \quad \text{[5]} \\
\sum \Sigma M_D &= 0; \quad F_A (0.3) + F_B (0.3) - F_D (0.3) = 0 \quad \text{[6]}
\end{align*}
\]

**Friction:** Assuming cylinder \( E \) slips at points \( C \) and \( D \) and cylinder \( F \) does not move, then \( F_C = \mu_C N_C = 0.5N_C \) and \( F_D = \mu_D N_D = 0.6N_D. \) Substituting these values into Eqs. [1], [2] and [3] and solving, we have

\[
\begin{align*}
N_C &= 377.31 \text{ N} \quad N_D = 188.65 \text{ N} \\
M &= 90.55 \text{ N} \cdot \text{m} = 90.6 \text{ N} \cdot \text{m} \quad \text{Ans}
\end{align*}
\]

If cylinder \( F \) is on the verge of slipping at point \( A, \) then \( F_A = \mu_A N_A = 0.5N_A. \) Substitute this value into Eqs. [4], [5] and [6] and solving, we have

\[
\begin{align*}
N_A &= 150.92 \text{ N} \quad N_B = 679.15 \text{ N} \quad F_B = 37.73 \text{ N}
\end{align*}
\]

Since \( (F_B)_{\text{max}} = \mu_B N_B = 0.5(679.15) = 339.58 \text{ N} > F_B, \) cylinder \( F \) does not move. Therefore the above assumption is correct.
8–134. The clamp is used to tighten the connection between two concrete drain pipes. Determine the least coefficient of static friction at A or B so that the clamp does not slip regardless of the force in the shaft CD.

**Free - Body Diagram.** Since member CA tends to move to the right, the frictional force \( F_A \) must act to the left as indicated on the free-body diagram of member CA shown in Fig. a.

**Equations of Equilibrium.** Referring to Fig. a,

\[ \Sigma F_x = 0: \quad F_{CA} \cos 68.20^\circ - F_A = 0 \quad F_A = 0.3714F_{CA} \]

\[ \Sigma F_y = 0: \quad F_{CA} \sin 68.20^\circ - N_A = 0 \quad N_A = 0.9285F_{CA} \]

To prevent slipping at A, the coefficient of static friction at A must be at least

\[ \mu_s = \frac{F_A}{N_A} = \frac{0.3714F_{CA}}{0.9285F_{CA}} = 0.4 \quad \text{Ans.} \]
If $P = 900\, \text{N}$ is applied to the handle of the bell crank, determine the maximum torque $M$ the cone clutch can transmit. The coefficient of static friction at the contacting surface is $\mu_s = 0.3$.

**SOLUTION**

Referring to the free-body diagram of the bellcrank shown in Fig. a, we have

$$\zeta + \Sigma M_B = 0; \quad 900(0.375) - F_C(0.2) = 0 \quad F_C = 1687.5\, \text{N}$$

Using this result and referring to the free-body diagram of the cone clutch shown in Fig. b,

$$\implies \Sigma F_x = 0; \quad 2\left(\frac{N}{2}\sin 15^\circ\right) - 1687.5 = 0 \quad N = 6520.00\, \text{N}$$

The area of the differential element shown shaded in Fig. c is

$$dA = 2\pi r\, ds = 2\pi r\frac{dr}{\sin 15^\circ} = \frac{2\pi}{\sin 15^\circ} r\, dr. \text{ Thus,}$$

$$A = \int_A dA = \int_{0.125\, \text{m}}^{0.15\, \text{m}} \frac{2\pi}{\sin 15^\circ} r\, dr = 0.08345\, \text{m}^2. \text{ The pressure acting on the cone surface is}$$

$$p = \frac{N}{A} = \frac{6520.00}{0.08345} = 78.13(10^3)\, \text{N} / \text{m}^2$$

The normal force acting on the differential element $dA$ is

$$dN = p\, dA = 78.13(10^3)\left[\frac{2\pi}{\sin 15^\circ}\right] r\, dr = 1896.73(10^3)r\, dr.$$

Thus, the frictional force acting on this differential element is given by

$$dF = \mu_s dN = 0.3(1896.73)(10^3)r\, dr = 569.02(10^3)r\, dr. \text{ The moment equation about the axle of the cone clutch gives}$$

$$\Sigma M = 0; \quad M - \int rdF = 0$$

$$M = \int rdF = 569.02(10^3) \int_{0.125\, \text{m}}^{0.15\, \text{m}} r^2\, dr$$

$$M = 270\, \text{N} \cdot \text{m} \quad \text{Ans.}$$
The lawn roller has a mass of 80 kg. If the arm BA is held at an angle of 30° from the horizontal and the coefficient of rolling resistance for the roller is 30 mm, determine the force $P$ needed to push the roller at constant speed. Neglect friction developed at the axle $A$, and assume that the resultant force $P$ acting on the handle is applied along arm $BA$.

**SOLUTION**

$$\theta = \sin^{-1} \left( \frac{30}{250} \right) = 6.892^\circ$$

$$\zeta + \Sigma M_O = 0; \quad -30(784.8) - P \sin 30^\circ(30) + P \cos 30^\circ(250 \cos 6.892^\circ) = 0$$

Solving,

$$P = 117.8 \text{ N}$$

Ans.
8.137. Two blocks A and B, each having a mass of 6 kg, are connected by the linkage shown. If the coefficients of static friction at the contacting surfaces are $\mu_A = 0.2$ and $\mu_B = 0.8$, determine the largest vertical force $P$ that may be applied to pin C without causing the blocks to slip. Neglect the weight of the links.

**Equations of Equilibrium:** From FBD (a),

\[ \Sigma F_x = 0; \quad T_y \cos 15^\circ - P \sin 45^\circ = 0 \quad T_y = 0.7321 P \]

\[ \Sigma F_v = 0; \quad T_A + 0.7321 P \sin 15^\circ - P \cos 45^\circ = 0 \]

\[ T_A = 0.5176 P \]

From FBD (b),

\[ \Sigma F_y = 0; \quad N_A - 0.5176 P \sin 45^\circ - 58.86 = 0 \quad [1] \]

\[ \Sigma F_x = 0; \quad 0.5176 P \cos 45^\circ - F_A = 0 \quad [2] \]

From FBD (c),

\[ \Sigma F_y = 0; \quad N_B - 0.7321 P \sin 60^\circ - 58.86 = 0 \quad [3] \]

\[ \Sigma F_x = 0; \quad F_B - 0.7321 P \cos 60^\circ = 0 \quad [4] \]

**Friction:** Assuming block A slips, then $F_A = \mu_A N_A = 0.2 N_A$. Substituting this value into Eqs. [1], [2], [3] and [4] and solving, we have

\[ P = 40.20 \text{ N} = 40.2 \text{ N} \]

\[ N_A = 73.575 \text{ N} \quad N_B = 84.35 \text{ N} \quad F_A = 14.715 \text{ N} \]

\[ T_B = 0.7321 P \]

Since \((F_A)_\text{max} = \mu_B N_B = 0.8(84.35) = 67.48 \text{ N} > F_A\), block B does not slip. Therefore, the above assumption is correct.
The uniform 60-kg crate \( C \) rests uniformly on a 10-kg dolly \( D \). If the front casters of the dolly at \( A \) are locked to prevent rolling while the casters at \( B \) are free to roll, determine the maximum force \( P \) that may be applied without causing motion of the crate. The coefficient of static friction between the casters and the floor is \( \mu_f = 0.35 \) and between the dolly and the crate, \( \mu_d = 0.5 \).

SOLUTION

Equations of Equilibrium: From FBD (a),

\[ + \sum F_y = 0; \quad N_d - 588.6 = 0 \quad N_d = 588.6 \text{ N} \tag{1} \]

\[ - \sum F_x = 0; \quad P - F_d = 0 \tag{2} \]

\[ \zeta + \sum M_A = 0; \quad 588.6(x) - P(0.8) = 0 \tag{3} \]

From FBD (b),

\[ + \sum F_y = 0; \quad N_B + N_A - 588.6 - 98.1 = 0 \tag{4} \]

\[ - \sum F_x = 0; \quad P - F_A = 0 \tag{5} \]

\[ \zeta + \sum M_B = 0; \quad N_A (1.5) - P(1.05) - 588.6(0.95) - 98.1(0.75) = 0 \tag{6} \]

Friction: Assuming the crate slips on dolly, then \( F_d = \mu_d N_d = 0.5(588.6) = 294.3 \text{ N} \). Substituting this value into Eqs. (1) and (2) and solving, we have

\[ P = 294.3 \text{ N} \quad x = 0.400 \text{ m} \]

Since \( x > 0.3 \text{ m} \), the crate tips on the dolly. If this is the case \( x = 0.3 \text{ m} \). Solving Eqs. (1) and (2) with \( x = 0.3 \text{ m} \) yields

\[ P = 220.725 \text{ N} \]

\[ F_d = 220.725 \text{ N} \]

Assuming the dolly slips at \( A \), then \( F_A = \mu_f N_A = 0.35 N_A \). Substituting this value into Eqs. (3), (4), and (5) and solving, we have

\[ N_A = 559 \text{ N} \quad N_B = 128 \text{ N} \]

\[ P = 195.6 \text{ N} = 196 \text{ N} \ (\text{Control!}) \]
The 3-Mg four-wheel-drive truck (SUV) has a center of mass at $G$. Determine the maximum mass of the log that can be towed by the truck. The coefficient of static friction between the log and the ground is $\mu_s = 0.8$, and the coefficient of static friction between the wheels of the truck and the ground is $\mu_s' = 0.4$. Assume that the engine of the truck is powerful enough to generate a torque that will cause all the wheels to slip.

**Free-Body Diagram.** Since the truck is about to move to the right, its driving force $F_t$ provided by the friction of all the wheels must act to the right as indicated on the free-body diagram of the truck shown in Fig. a. Here, $F_t$ is required to be maximum, thus $F_t = \mu_s'(N_A + N_B) = 0.4(N_A + N_B)$. Since the log is required to be on the verge of sliding to the right, the frictional force $F_f$ must act to the left such that $F_f = \mu_s N_l = 0.8N_l$.

**Equations of Equilibrium.** Referring to Fig. a, we have

\[
\begin{align*}
\sum F_y &= 0; \\
N_A + N_B - 3000(9.81) &= 0 \\
N_A + N_B &= 29430 \text{ N} \\
\sum F_x &= 0; \\
0.4(29430) - T &= 0 \\
T &= 11772 \text{ N} \\
\sum M_B &= 0; \\
N_A(2.8) + 1772(0.5) - 3000(9.81)(1.6) &= 0 \\
N_A &= 14715 \text{ N} > 0 \text{ (OK!)}
\end{align*}
\]

Using the result of $T$ and referring to Fig. b, we have

\[
\begin{align*}
\sum F_y &= 0; \\
N_l - m_l(9.81) &= 0 \\
N_l &= 9.81m_l \\
\sum F_x &= 0; \\
1772 - 0.8(9.81m_l) &= 0 \\
m_l &= 1500 \text{ kg} \quad \text{Ans.}
\end{align*}
\]
A 3-Mg front-wheel-drive truck (SUV) has a center of mass at \( G \). Determine the maximum mass of the log that can be towed by the truck. The coefficient of static friction between the log and the ground is \( \mu_s = 0.8 \), and the coefficient of static friction between the front wheels of the truck and the ground is \( \mu'_s = 0.4 \). The rear wheels are free to roll. Assume that the engine of the truck is powerful enough to generate a torque that will cause the front wheels to slip.

**Free-Body Diagram.** Since the truck is about to move to the right, its driving force \( F_A \) provided by the friction of the front wheels must act to the right as indicated on the free-body diagram of the truck shown in Fig. a. Here, \( F_A \) is required to be maximum, so that \( F_A = \mu'_s N_A = 0.4 N_A \). Since the log is required to be on the verge of sliding to the right, the frictional force \( F_I \) must act to the left such that \( F_I = \mu_s N_I = 0.8 N_I \).

**Equations of Equilibrium.** Referring to Fig. a, we have

\[
\begin{align*}
\sum F_x &= 0; & \quad 0.4 N_A - T &= 0 \\
\sum F_y &= 0; & \quad T(0.5) + N_A (2.8) - 3000(9.81)(1.6) &= 0
\end{align*}
\]

Solving Eqs. (1) and (2) yields

\[
N_A = 15696 \text{ N} \quad T = 6278.4 \text{ N}
\]

Using the result of \( T \) and referring to Fig. b, we have

\[
\begin{align*}
\sum F_y &= 0; & \quad N_I - m_I (9.81) &= 0 \\
\sum F_x &= 0; & \quad 6278.4 - 0.8(9.81m_I) &= 0
\end{align*}
\]

\( m_I = 800 \text{ kg} \)  \( \text{Ans.} \)
A roofer, having a mass of 70 kg, walks slowly in an upright position down along the surface of a dome that has a radius of curvature of \( r = 20 \text{ m} \). If the coefficient of static friction between his shoes and the dome is \( \mu_s = 0.7 \), determine the angle \( \theta \) at which he first begins to slip.

\[
\begin{align*}
\sum F_x &= 0; \quad N - 70(9.81) \cos \theta = 0 \\
\sum F_y &= 0; \quad 70(9.81) \sin \theta - 0.7N = 0
\end{align*}
\]

Solving Eqs. (1) and (2) yields:

\[ \theta = 35.0^\circ \quad \text{Ans} \]

\[ N = 562.6 \text{ N} \]
Determine the minimum horizontal force $P$ required to hold the crate from sliding down the plane. The crate has a mass of 50 kg and the coefficient of static friction between the crate and the plane is $\mu_s = 0.25$.

**SOLUTION**

_Free-Body Diagram:_ When the crate is on the verge of sliding down the plane, the frictional force $F$ will act up the plane as indicated on the free-body diagram of the crate shown in Fig. a.

_Equations of Equilibrium:_

\[
\sum F_x = 0; \quad N - P \sin 30^\circ - 50(9.81) \cos 30^\circ = 0
\]

\[
\sum F_y = 0; \quad P \cos 30^\circ + 0.25N - 50(9.81) \sin 30^\circ = 0
\]

Solving

\[
P = 140 \text{ N}
\]

\[
N = 494.94 \text{ N}
\]

_Ans._
Determine the minimum force $P$ required to push the crate up the plane. The crate has a mass of 50 kg and the coefficient of static friction between the crate and the plane is $\mu_s = 0.25$.

**SOLUTION**

When the crate is on the verge of sliding up the plane, the frictional force $F'$ will act down the plane as indicated on the free-body diagram of the crate shown in Fig.6.

\[ \sum F_x = 0; \quad N' - P \sin 30^\circ - 50(9.81) \cos 30^\circ = 0 \]
\[ \sum F_y = 0; \quad P \cos 30^\circ - 0.25N' - 50(9.81) \sin 30^\circ = 0 \]

Solving,

\[ P = 474 \text{ N} \]
\[ N' = 661.92 \text{ N} \]
A horizontal force of $P = 100 \text{ N}$ is just sufficient to hold the crate from sliding down the plane, and a horizontal force of $P = 350 \text{ N}$ is required to just push the crate up the plane. Determine the coefficient of static friction between the plane and the crate, and find the mass of the crate.

**SOLUTION**

**Free-Body Diagram:** When the crate is subjected to a force of $P = 100 \text{ N}$, it is on the verge of slipping down the plane. Thus, the frictional force $F$ will act up the plane as indicated on the free-body diagram of the crate shown in Fig. a. When $P = 350 \text{ N}$, it will cause the crate to be on the verge of slipping up the plane, and so the frictional force $F'$ acts down the plane as indicated on the free-body diagram of the crate shown in Fig. b. Thus, $F = \mu_s N$ and $F' = \mu_s N'$.

**Equations of Equilibrium:**

\[ +\Sigma F_x = 0; \quad N - 100 \sin 30^\circ - m(9.81) \cos 30^\circ = 0 \]

\[ +\Sigma F_y = 0; \quad \mu_s N + 100 \cos 30^\circ - m(9.81) \sin 30^\circ = 0 \]

Eliminating $N$,

\[ \mu_s = \frac{4.905m - 86.603}{8.496m + 50} \]

Also by referring to Fig. b, we can write

\[ +\Sigma F_x = 0; \quad N' - m(9.81) \cos 30^\circ - 350 \sin 30^\circ = 0 \]

\[ +\Sigma F_y = 0; \quad 350 \cos 30^\circ - m(9.81) \sin 30^\circ - \mu_s N' = 0 \]

Eliminating $N'$,

\[ \mu_s = \frac{303.11 - 4.905m}{175 + 8.496m} \]

Solving Eqs. (1) and (2) yields

\[ m = 36.5 \text{ kg} \]

\[ \mu_s = 0.256 \]

**Ans.**

**Ans.**